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W. J. Gage & Co.'s Mathematical Series

HANDBOOK

TO THE

NEW HAMBLIN SMITH'S ARITHMETIC

BY

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DEPUTY MINISTER OF EDUCATION FOR MANITOBA

PRICE, $1.50

W. J. GAGE & CO., LIMITED
TORONTO

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PREFACE

THIS Handbook consists of three parts: Part I. gives the answers to the exercises, Part II. consists of solutions of the more difficult problems, and Part III. is a supplement giving additional information and discussing some topics which were omitted in the text-book.

The work has been prepared for the use of teachers and private students. With the multiplicity of duties ordinarily devolving on the teacher, time cannot always be had to solve all questions that may be presented by the pupil. Hence, a work such as this becomes a great convenience, if not an actual necessity.

It has not been thought best to solve such exercises as are comparatively easy or merely mechanical. Hence, many of those under Fractions, Extraction of Roots, Compound Numbers, Interest, etc., are omitted.

The solutions have in general, been given with strict reference to the Unitary Method, thus showing its applicability to questions of every variety and degree of difficulty. They do not exhibit all the calculations at large, but they furnish results which serve to verify the operations at the successive stages of the process. In this way all that is necessary has been brought within a narrow compass, and the connection of the different parts of each solution will be more readily perceived.

It has not been the aim of the authors to make a mere Key, but to exhibit the best and neatest mode of working Arithmetical Exercises. Not only are neatness and method encouraged by the habit of arranging figures in their exact places, but the accuracy of the answer is best secured by the same means.

Indications of any errors or obscurities will be thankfully received.

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HANDBOOK

TO THE

NEW HAMBLIN SMITH'S ARITHMETIC

PART I.—ANSWERS

Exercise I. Page 11

1. 83706951. 2. 94114466. 3. 77480828.
4. 147945794. 5. 121964686. 6. 136964779.
7. 33548. 8. 26678. 9. 13504.
10. 3,649,280,169,549. 12. 128; 116.
13. 185. 14. 691896. 15. 75.
16. 113. 18. 278. 19. 522; 2604; 159.
21. $75; $27000. 22. 37 da.
23. 8822 bu. wheat; 1178 bu. oats. 24. 1200 yd.
27. 9900; Rem. 100. 28. 27 hr. 29. 5504940730.
30. 619161890. 31. 7283. 32. $14541.
33. 7684 and 978. 34. 72 da. 35. 252.
36. 296237. 37. $13300, $11900, $10500.
38. 7000. 39. 450 lb. 40. 31239.
41. $232. 42. $30. 44. 9999000025.
45. $37569. 46. 11796 steps. 47. $21000; $5400.
49. 170680900742874252. 50. 2796203.
51. 786543. 52. 31116. 53. 39.
54. 689000. 55. 123; 7317. 56. 496; [illegible].
57. 9801, 998001, 99980001, 9999800001, 9604, 996004, 99960004,
9999600004, 9409, 994009, 99940009, 9999400009.
58. 623309. 59. 37632. 60. 24725.
61. 377. 62. 567761. 63. $210.
64. $7\frac{9}{13}$ mi. 65. 77190.

66. 42238274625; $1959\frac{22}{85}$. 67. 90 mi.
68. 400 mi. 69. $115. 70. 5*s*.
71. 90 da. 72. 3998936616. 73. *B*.
74. 478; 369; 584 75. $46850. 76. $22950.
77. 37199. 78. Min. 921654; Sub. 184796.
79. Div. 561; Q. 943. 80. 1071 lb.; $52.92. 81. 1980 lb.; $59.40.

ORAL EXERCISE. PAGE 20

1. $10.08. 2. 1995 lb. 3. 15840 yd.
4. 10080 min. 5. 8. 6. 1728 mi.
7. 234 mi. 8. 945 ~~mi.~~ 9. 100 gal.
10. 4957 sheep. 11. 365. 12. $90.
13. $24. 14. 18c. 15. 48 hr.
16. 125 pages. 17. 135 bags. 18. 170.
19. 1. 20. 1860; 2114. 21. 960 mi.
22. 64.

EXERCISE II. PAGE 22

4. 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 180, 360.
5. 97. 7. 7. 8. 130.
9. 21, 22, 23, 24. 12. 1, 3, 7, 9. 13. 203.
14. 3366000. 15. 46, 47, 48, 49, 50. 17. 27, 29, 31, 33, 35.
18. 3391 and 2699 are prime; 14787 and 1477 are composite.
19. 14, 21, 35.
20. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2026, 2700, 4050, 8100.
21. 76. 22. 189. 23. 20.
24. 15. 25. 55 sacks. 26. $$3.12\frac{1}{2}$.
27. $5\frac{1}{4}$c. 28. 888030. 29. 15 pieces.
30. 12 bbl. 31. 30 mi. 32. 24 firkins.
33. 233. 34. 7. 36. 25.
37. 7 oz. 38. 23 apples; 89 oranges.
39. 1856, 4466. 40. 44496 rails.
41. Bags of 1, 2, or 3 bu. each.; bins of 300, 200, or 15 bu. each.

42. 40 gr.
43. 446, 475.
44. 4234.
45. 2520.
46. 287.
47. 123328.
48. 420.
49. 3327.
50. 35 times.
51. 7.
52. 36, 84, 132.
54. $1650.
55. 60 min.
56. 982832.
57. 29.
58. 60 hr.; *A*, 300 mi.; *B*, 240 mi.; *C*, 180 mi.
59. $9\frac{3}{4}$ mi.
60. 70560.
61. 60.
62. 3 and 6.
63. 44 times; 9284 trees.
64. 12 lb.
65. 1474 marbles.
66. 840 sec.
67. *B*; 16 yd.
68. 460800.
69. 32 times.
70. 30; 360; 36, 40, 45, 60, 72, 90, 120, 180.
71. 28 bu.
72. 35 mi.
73. 7113120 da.
74. *A*, 21 mi.; *B*, 14 mi.; *C*, 12 mi.; *D*, $10\frac{1}{2}$ mi.
75. 900.
76. 2521777.
77. $1152.

ORAL EXERCISE. PAGE 30

1. 10080.
2. 5, 5, 11, 11.
3. 144.
4. 90.
5. 210.
6. 35 ft.
7. 97.
8. $480.
9. 24.
10. $56\frac{1}{4}$.
11. 75.
12. 96 ft.
13. 300 boards.
14. 480 ac.
15. 50 bags.
16. 175 bu.
17. 3 min.
18. 12 rd.
19. 42.
20. 1 qt., 2 qt., 3 qt., 4 qt., 6 qt., 12 qt.

EXERCISE III. PAGE 33

1. $\frac{1358}{15}$, $\frac{6389}{13}$, $\frac{249088}{409}$.
2. $3\frac{477}{1000}$, $13\frac{51}{73}$, $17\frac{292}{709}$.
3. $\frac{63}{81}$, $\frac{14}{23}$.
4. $\frac{3}{4}$, $\frac{29}{517}$, $\frac{9}{13}$.
5. $\frac{408}{714}$, $\frac{630}{714}$, $\frac{364}{714}$, $\frac{455}{714}$; $\frac{180}{420}$, $\frac{340}{420}$, $\frac{348}{420}$, $\frac{161}{420}$, $\frac{282}{420}$.
6. In descending order, $\frac{13}{15}$, $\frac{9}{11}$, $\frac{17}{21}$; $\frac{7}{33}$, $\frac{9}{43}$, $\frac{11}{53}$.
7. (1) $1\frac{179}{196}$. (2) $25\frac{41}{144}$. (3) $26\frac{18}{25}$.
8. (1) $\frac{13}{418}$. (2) $\frac{218}{1911}$. (3) $10\frac{25}{66}$. (4) $53\frac{13}{16}$.
9. (1) $\frac{135}{308}$. (2) $\frac{37}{228}$. (3) $\frac{32}{405}$. (4) 225.
10. (1) $\frac{2}{5}$. (2) $\frac{49}{54}$. (3) $9\frac{1}{2}$. (4) $1\frac{5}{6}$.
11. (1) $1\frac{1}{3}$. (2) $1\frac{37}{44}$. (3) $18\frac{4}{7}$.

12. (1) $\frac{1}{34}$. (2) $\frac{1}{54600}$.
13. (1) 285. (2) $8350\frac{1}{3}$. (3) 24.
14. (1) $\frac{1}{2}$. (2) $\frac{685}{756}$. (3) $14\frac{42}{193}$. (4) $\frac{75}{77}$. (5) 1. (6) $\frac{1}{2}$.

EXERCISE IV. PAGE 36

1. $\$49\frac{1}{5}$. 2. $5\frac{82}{157}$. 3. \$13860.
4. $3\frac{27}{70}$ and $\frac{57}{70}$. 6. $\frac{23}{44}$, $\frac{20}{55}$, $\frac{7}{15}$, $\frac{8}{35}$. 7. $\frac{708}{1000}$.
9. Ship, \$24000; cargo, \$36000. 11. $\$18\frac{2}{3}$.
13. $\frac{473}{18500}$. 14. *A*, 20; *B*, 48; *C*, 84.
16. $\frac{27}{35}$. 17. $14\frac{9}{17}$.
18. Horse, \$120; carriage, \$105; harness, \$25.
19. *A*, \$4334; *B*, \$1474; *C*, \$3080. 21. $3\frac{22}{39}$.
22. \$2015; 465 sheep; 390 calves; 806 pigs.
23. \$180. 24. 18 ft. 26. $\frac{3}{15}$, $\frac{5}{15}$, $\frac{7}{15}$.
27. 1. 28. \$40. 29. $\frac{5}{36}$ ft.
31. $\$1333\frac{1}{3}$; $\frac{1}{30}$. 32. 1000000.
33. 30 min.; *A*, 6 times; *B*, 5 times; *C*, 4 times.
34. *A*, $\frac{5}{36}$; *B*, $\frac{5}{28}$, *C*, $\frac{10}{231}$; *D*, $\frac{15}{77}$. 36. $\frac{1}{6}$; $\frac{8}{91}$.
37. 36. 38. 30 min.; 4500 rd.; 3600 rd.; 3000 rd.
39. 252. 40. 72 ft.; 64 ft. 41. $10\frac{1}{4}$ bu. 42. $1\frac{3}{7}$.
43. 1350 men; Irish, 540; Scotch, 405; English, 405. 44. \$2520.

EXERCISE V. PAGE 40

2. (*c*) .003; .0019; 172.95.
3. (1) 8741.2062. (2) .001. (3) 26.841.
4. (*e*) (1) .0041588. (2) 18.9326. (3) 1.01.
5. (1) 457.61. (2) .00192. (3) 0.000926. (4) $100.01\dot{6}$.
6. (1) .44. (2) $.21\dot{6}$. (3) $.\dot{8}5714\dot{2}$. (4) $2.3\dot{4}\dot{5}$.
7. (1) $.\dot{4}\dot{2}$. (2) $.4\dot{7}$. (3) $2.4\dot{3}04\dot{7}$. (4) $124.79\dot{2}867\dot{3}$.
8. $\frac{5}{32}$, finite; $\frac{7}{12}$, mixed repetend; $\frac{19}{24}$, mixed repetend; $\frac{17}{25}$, finite, $\frac{47}{56}$, mixed repetend; $\frac{5}{13}$, pure repetend; $\frac{7}{26}$, mixed repetend.
9. (1) $\frac{5}{111}$. (2) $\frac{6}{11111}$. (3) $\frac{421}{990}$. (4) $7\frac{2009}{9990}$. (5) $4\frac{57}{2250}$. (6) $18\frac{36053}{99990}$.
10. (1) $15.8\dot{4}3\dot{0}$. (2) $1.7\dot{7}8005\dot{2}$. (3) $10.60\dot{9}7078\dot{8}$.
11. (1) 20.758698 ... (2) .00325. (3) $.\dot{8}401\dot{0}$. (4) $.12\dot{3}$.

Exercise VI. Page 43

3. 1st. 4. 263 times; $\frac{1}{3}$.
5. .0000006 and .0000009. 6. $14.90.
7. 3.715. 8. $1\frac{2}{9}$. 9. .7142.
11. $$816\frac{2}{3}$. 12. 13; $3000. 13. $$232\frac{2}{3}$.
14. $\frac{13200}{2009}$. 15. $21.60. 16. 425.
17. $8.75. 18. $82\frac{1}{4}$ yd. 20. $\frac{39371}{2534400}$.
21. 10.7608 mi. 22. $13\frac{23}{48}$.
23. *A*, $$192.23\frac{1}{2}$; *B*, $$145.53\frac{1}{2}$; *C*, $$110.94\frac{1}{2}$.
25. $$34\frac{1}{3}$. 26. 7.025, .17141.

Exercise VII. Page 49

1. .09484. 2. .002521. 3. 165.6995.
4. 235.104. 5. 26.38702. 6. 1.611.
7. .0374. 8. 426.104. 9. 170.3367.
10. .928.

Exercise VIII. Page 49

1. $4200. 2. $2691\frac{13}{17}$ yd. 3. $2000\frac{10001}{500000}$.
4. $.4\dot{3}\dot{2}$. 5. .0189 . . . 6. $$3733\frac{1}{3}$.
7. $$9.37\frac{1}{2}$. 8. $12705. 9. .75.
10. 14. 11. $4\frac{1}{21}$. 12. 46.
13. 1520 t. 14. 10861578 nearly. 15. .21; 2100; $\frac{6667}{3630}$.
16. 9000 men. 17. 42000 men. 18. .4 in.
19. $\frac{6}{37}$. 20. 1. 21. 937; .02268 in.
22. 7. 23. 2.198 in.
24. 345 boys; 15 masters. 26. $4\frac{1}{2}$ mi.
27. $1\frac{1}{6}$ mi. 28. .05. 29. 2*s*. 6*d*.
30. 5 hr. 31. 54 sheep. 32. $\dfrac{27\frac{1}{2}}{71\frac{1}{2}}$.
33. 8085. 34. 36400. 35. 45 ft. and 50 ft.
36. $1575. 37. 6 yr.
38. Between .0001 and .0002.

ORAL EXERCISE. PAGE 53

1. $\frac{17}{3}$; $\frac{43}{8}$.
2. $\frac{43}{9}$; $\frac{47}{7}$.
3. $\frac{24}{20}$; $\frac{13}{30}$.
4. $\frac{9}{12}$, $\frac{8}{12}$, $\frac{10}{12}$.
5. $\frac{5}{6}$.
6. $4.
7. $24.
8. 35 pints.
9. $26.
10. $46.
11. 40 lots.
12. $\frac{8}{12}$; $\frac{9}{12}$.
13. $\frac{5}{6}$; $1\frac{1}{12}$.
14. $$1\frac{1}{4}$.
15. 36.
16. $\frac{11}{12}$.
17. $\frac{1}{8}$.
18. 8 ft.
19. $\frac{13}{20}$ ac.
20. $40.
21. 60 candidates.
22. $$2\frac{17}{20}$.
23. 84.
24. $\frac{11}{12}$.
25. $\frac{5}{12}$.
26. $\frac{1}{8}$.
27. $3.15.
28. 36 times.
29. 7 mi.
30. $\frac{5}{4}$.
31. 12.
32. $7\frac{3}{5}$ da.
33. $$3\frac{1}{10}$.
34. $5\frac{1}{7}$ rd.
35. 24 apples.
36. $$22\frac{1}{2}$.
37. $200.
38. $$12\frac{1}{2}$.

EXERCISE IX. PAGE 56

1. (1) 497025. (2) .000256. (3) $68\frac{1}{16}$.
2. (1) 350402625. (2) .000004096. (3) $561\frac{33}{64}$.
3. (1) 247033850625. (2) .000000065536. (3) $4632\frac{129}{256}$.
4. 16200; 3673.
5. 47.11.
6. 9261000.
7. .03.
9. 613089; 616225.

EXERCISE X. PAGE 59

1. (1) 3. (2) 4. (3) 6.
2. 14.
3. 23.
4. 32.
5. 75.
6. 297.
7. 345.
8. 327.
9. 867.
10. 440.
11. 835.
12. 6031.
13. 4698.
14. 23456.
15. 72500.
16. 2031.
17. 739000.
18. 5678.
19. 437962.
20. 17325.

EXERCISE XI. PAGE 59

1. 4.1.
2. .95.
3. .51.
4. .25.
5. .027.
6. 131.31.
7. 1.001.
8. 210.75.
9. 137.65.

EXERCISE XII. PAGE 60

1. 4.4721.
2. 5.4772.
3. .9486.
4. .3478.
5. .4110.
6. .1264.
7. .0252.
8. .0347.
9. 4.0305.
10. .9999.
11. .5025.
12. 6.4833.

EXERCISE XIII. PAGE 60

1. $\frac{6}{7}$.
2. $\frac{8}{11}$.
3. $\frac{17}{25}$.
4. $\frac{37}{87}$.
5. $\frac{123}{427}$.
6. $2\frac{1}{4}$.
7. $2\frac{2}{3}$.
8. $1\frac{10}{13}$.
9. $8\frac{1}{9}$.
10. $6\frac{1}{3}$.
11. $4\frac{1}{3}$.
12. $3\frac{2}{7}$.
13. .7905.
14. .6454.
15. 2.5298.
16. 3.0822.
17. 8.7649.
18. 4.0632.

EXERCISE XIV. PAGE 61

1. 855.
2. 797.
3. 2223.
4. 748; 187.
5. 405; 270.
6. 12.96; $\frac{23}{49}$.
7. 175.25; $\frac{25}{49}$; .000365.
8. 900.
9. 45.
10. 4007.
11. 126.
13. 883.
14. 950.12\frac{1}{2}$.
15. 455.368 ft.
16. 103.66; $\frac{8}{9}$.
17. 217$\frac{10}{11}$ lb.
18. 590.05 … yd.
19. $5.28.
20. 742.
21. 20 ft. by 20 ft.
22. 190$\frac{16}{35}$.
23. 27.202 … ft.

ORAL EXERCISE. PAGE 63

1. 41.
2. 42. in.
3. 220 in.
4. 96 yd.
5. 625 sq. yd.
6. 20 ft. by 20 ft.
7. 25.
8. 5.
9. $2560.
10. 75 yd. by 25 yd.
11. 15 ft.
12. 256 sq. ft.
13. 5 in.
14. 30.
15. 7.
16. $\frac{12}{35}$.
17. 5.
18. 35.
19. 65.
20. 12 ft.
21. 7.
22. 16 yd.
23. 6 ft. sq.
24. $63.
25. 168 yd. by 56 yd.
26. 90.
27. 45.

EXERCISE XV. PAGE 65

1. 4 cwt. $61\frac{1}{4}$ lb.
2. $\frac{613}{2016}$.
3. $\frac{497}{957}$.
4. 13*s.* 6*d.*
5. $\frac{3}{11}$.
6. £16 0*s.* 6*d.*; 93 lb. 12 oz.
7. .3285.
8. 86 rd. 4 yd. $1\frac{1}{2}$ in.
9. $\frac{1}{3}$.
10. 4 t. 16 cwt. 17.4 lb.
11. $.1\dot{1}9047\dot{6}$.
12. $.002\dot{7}$.
13. $\frac{2880}{17}$.
14. $2488\frac{1}{3}$ gr.
15. $.5\dot{3}$.
16. 18° 25′ $16\frac{7}{20}$″.
17. 6.
18. 45 mi.
19. $\frac{321}{352}$.
20. 21 yd. 2 ft. $2\frac{3}{8}$ in.
21. 60 min.
22. 1.2535 ... lb.
23. $7442\frac{2}{35}$ lb.
24. 56 yd.
25. £9 3*s.* 6*d.*
26. 2*s.* $5\frac{7}{10}$*d.*
27. 90.
28. 800000.
29. 2.1723; .065169 t.
30. 6 oz.
31. 40 : 53.
32. $366\frac{6}{59}$.
33. 3 da. 2 hr. 20 min.
34. 633603 in.
35. $13\frac{1307}{1321}$ ft.
36. 1.49 p.m.; $149\frac{3}{4}$ mi.
37. 19 mi. $1464\frac{27}{32}$ yd.
38. 7488 strokes.
40. 3600; $36.
41. *A*, 3 ac. 1 ro. 20 per. 21 yd. $77\frac{1}{4}$ in.; *B*, 6 ac. 3 ro. 1 per. 11 yd. 7 ft. $118\frac{3}{4}$ in.; *C*, 7 ac. 2 ro. 16 per. 17 yd. 1 ft. $29\frac{3}{4}$ in.
42. 29 yd.
43. 16 t. 4 cwt.; 10 cwt. 80 lb.
44. $$11.37\frac{1}{2}$.
45. 259 bu. 2 pk. 1 gal. $1\frac{193}{231}$ pt.
46. $96.93...
47. Loses $2.
48. $13\frac{43}{64}$c.
49. 20 gr.
50. 4 ac. 2 ro. 7 po. 3 yd.; 699 ac. 3 ro. 13 po. $8\frac{1}{4}$ yd.
51. 2 hr. 10 min.
52. Loses £295 3*s.* $2\frac{1}{2}$*d.*

ORAL EXERCISE. PAGE 70

1. 134 pt.
2. $19\frac{1}{2}$ oz.
3. 23 da.
4. 1856, 1892, 1904.
5. 5 ft. 6 in.
6. 43 ft. 6 in.
7. $\frac{1}{2}$ as far.
8. $1.54.
9. $6\frac{1}{4}$ cd.
10. $90.
11. 54.
12. $28.70.
13. $16.
14. 99c.
15. 297 steps.
16. $21\frac{1}{3}$ sq. yd.
17. 32 cu. ft.
18. 42 bottles.
19. $13.90.
20. $2.90.
21. 9 lb.
22. 11 t. 14 cwt.
23. 70 doz.
24. 2 cwt. 15 lb.
25. 2 t. 11 cwt.
26. 3 bu. 1 pk.
27. $4.90.
28. 27 lb. 12 oz.

EXERCISE XVI. PAGE 74

2. 5007.039 m. 3. 17800.308 m. 4. 60070.807 m.
5. 5760 m.; 57600 dm.; 5760000 mm.
6. 8.769 m.; .08769 Hm.; .008769 Km.
9. 5887.816 m. 10. 2999.494 m. 11. 119.73 Km.
12. $2180.40. 13. 3100 posts. 14. 2008 m.
15. $3230\frac{10}{33}$ times. 16. 24.7824 Km.; 247824 dm.
17. $46. 18. 17.45 m.

EXERCISE XVII. PAGE 76

1. 5.0205 Ha.; 502.05 a.; 50205 ca. 2. 19070005 sq. m.
3. 607.81 sq. m. 4. 2.705608 sq. m. 9. 134.82 m.
10. 28.415 ... m. 11. $1136.25. 12. 9900 ca.
13. 786.96 Ha. 14. $14000. 15. 12470.30 sq. m.

EXERCISE XVIII. PAGE 77

1. 715.007078 cu. m.; 715007.078 cu. dm.; 715007078 cu. cm.
2. 7047.08 l.; 7047080 ml.
3. .45678 cu. Dm.; 456780 cu. dcm. 4. $22.33.
5. 899.64 st. 6. 253.98 st. 7. $22.53 ...
10. 157.3 loads. 11. 7680 l. 12. 3 times.
13. $8.89. 14. $12.69. 15. $7.424.

EXERCISE XIX. PAGE 79

2. 345.65 g.; 3.4565 Hg.; .34565 Kg. 3. 430357 cg.
4. 6230 g. 5. 37.005 Kg.
6. 710500 g.; 710500000 mg.; 7105000 dg. 7. 4500 Kg.
8. 840 Kg. 9. 780 Kg. 10. 900 letters.
11. $3.04. 12. 7882.4 Kg. 13. 229.5 Kg.
14. .001293 to 1. 15. 4 to 5.

EXERCISE XX. PAGE 80

1. 1067.25 dm. 2. 15000 mg. 3. 43.7 mm.; 4.37 cm.
4. 155000 sq. cm. 5. 1086.42 sq. dcm. 6. 1725 g.

7. 100 mg.; 10000 dg. 8. 256700 cg.
9. 5000 mg. 10. 567.875 cu. cm. 11. 3720 l.
12. 24855.303 mi. 13. 5168240 g. 14. .393696 in.
15. 111835$\frac{2}{7}$ m. 16. 1609.306 m. 17. 27951 : 12500.
18. £84. 19. 2.20 lb. 20. 15.495 … mi.
21. .072507 Kg. 22. 4545 cu. cm. 23. $3.29 …
24. 1.452 Kg. 25. 7911.5 … mi. 26. 13.5 Kg.
27. $\frac{4}{9}$ m. 28. 5.005005 st. 29. .03531 … cu. ft.
30. 100000 l. 31. 6000 cu. m. 32. 8$\frac{1}{3}$ m.
33. 68.1 m. 34. 140 ca. 35. $61.25.
36. $909.\dot{0}\dot{9}$ Kg. 37. 1 mm. 38. 23684$\frac{4}{19}$ sec.
39. 28.316 … l. 40. 1000 : 1840. 41. 905.6 Kg.
42. 80 l. 43. 2194.50 Hl. 44. 5906.25 Kg.
45. $81.777.

Oral Exercise. Page 84

2. .001. 3. 10 m.
4. 78000 dcm.; 4860 mm. 5. 78.9 l.
6. 84.568 cu. m. 7. 8 cu. m.
8. 463.68 sq. Dm.; 46368000000. sq. mm. 9. .045786.
10. 370 cm. 11. 15 Ha.; 150000 ca.
12. 1000000 cu. cm. 13. 100000 cu. cm. 14. 1000 l.
15. Same. 16. 1000000 g. 17. 1000000 mg.
18. 15$\frac{3}{8}$c. 19. 3 m.; 3.13 m.; 14.02 m.; 20.50 m.
20. .01. 21. 1 Kg. = 10000 dg.
22. 2080 g.; 27.05 g.; .367 g.; 7005 g. 23. 512638 cm.
24. 566.99 … g. 28. 14 times.
29. 2.31 m.; 285600 dcm. 30. 343000 l.
31. 196.7 st. 33. 425.095047 cu. m.; .000000005 cu. m.
34. 254757 ca. 35. 9830.7 Dl. 36. 70c.
37. $4.998. 38. 478 times. 39. .156 Ha.

Exercise XXI. Page 87

1. 1 hr. 40 min. 56 sec. 2. 9 min. 52 sec.
3. 3 hr. 9 min. 4. 1 hr. 3 min. 48 sec. 5. 6 hr. 6 min. 8$\frac{1}{3}$ sec.
6. 7 hr. 9 min. 28 sec. 7. 6 hr. 10 min. 56 sec.

8. 3 hr. 38 min. $51\frac{2}{3}$ sec. a.m.
9. 7 hr. 38 min. 48 sec.
10. 8 hr. 54 min. 32 sec. p.m.
11. 56° 24′.
12. 9° 20′.
13. 8° 28′.
14. 39° 55′.
15. 18° 48′.
16. 123° 5′.
17. 104° 37′.
18. 34° 26′ 45″ E.
19. 106° W.
20. 13° 23′ 43.2″ E.
21. 94° 32′ W.
22. 75° 42′.
23. 4 hr. 26 min. 26 sec. p.m.
24. 5 hr. 44 min. p.m.
25. 19 min. 36 sec.; Standard
26. 15 min. 8 sec.; Quebec.
27. 115° 35′.
28. (*a*) 1 hr. 20 min. (*b*) 1 hr. 40 min. (*c*) 2 hr. 24 min. (*d*) 3 hr. 12 min.
29. 8 hr. 47 min. 55 sec. p.m.
30. 97° 7′.

ORAL EXERCISE. PAGE 90

1. 27°.
2. 75th E.
3. 3 hr. 44 min.
4. 11° 52′ 30″.
5. 75° 55′.
6. 2 hr. 2 min. p.m.
7. 56°.
8. 18° 45′; east.
9. 34° 5′.
10. 70°.
11. 6° 15′; east.
12. 7 hr. 12 min. p.m.
13. 5 hr. 31 min. 32 sec. a.m.
14. 6 hr. 8 min.
15. (*a*) 1 hr. 36 min. p.m. (*b*) 5 hr. 8 min. a.m. (*c*) 4 hr. 24 min. a.m. (*d*) 4 hr. 24 min. p.m.
16. (*a*) 41° 15′. (*b*) 51°. (*c*) 64° 30′. (*d*) 100° 30′.

EXERCISE XXII. PAGE 92

1. 56 da.
2. 45 men.
3. 75c.
4. 6 da.
5. $1\frac{1}{2}$ da.
6. $47775.
7. 25 men.
8. $$543\frac{21}{47}$.
9. 203 mi.
10. $3240.
11. $$29\frac{1}{13}$.
12. 54 da.
13. 20c.
14. $158.95.
15. 7 hr.
16. $61\frac{1}{3}$ ac.
17. 25 horses.
18. 12 hr.
19. $$408\frac{1}{3}$.
20. $9.15.
21. $14.40.
22. $$7833.33\frac{1}{3}$.
23. $5040.
24. 18.7 lb.
25. $\frac{14}{165}$.
26. 27 min.
27. $$1182.12\frac{1}{2}$.
28. $286\frac{2}{13}$ mi.
29. $400.
30. £1 17*s.* 4.128*d.*
31. $18\frac{3}{4}$ cwt.
32. 14.208 rolls.

33. \$160953.94; 181.527 a.
34. 6 horses.
35. $21\frac{1}{3}$ da.
36. 480 ac.
37. \$1152.
38. 11268 soldiers.
39. 4000 horses.
40. \$10.52.
41. \$54.61$\frac{1}{4}$.
42. \$360.
43. .16 bu.
44. 20 da.
45. \$78.60.
46. 27 laborers.
47. 75 burners.
48. 6 wk.
49. \$396.
50. 7 wk.
51. 4 da.
52. $155\frac{3}{4}$ qr.
53. $5\frac{1}{2}$ wk.
54. 84 da.
55. 10 da.
56. 2 da.
57. $3\frac{9}{17}$ da.
58. $3\frac{3}{5}$ hr.
59. $13\frac{37}{191}$ da.
60. $2\frac{61}{148}$ da.
61. $2\frac{22}{73}$ da.
62. 6 hr.
63. $9\frac{3}{13}$ min.
64. 120 min.
65. $2\frac{2}{3}$ min.
66. 10 da.
67. 4 hr.
68. 18 da.
69. $\frac{29}{36}$.
70. *A*, $\frac{81}{94}$; *B*, $1\frac{17}{47}$ da.; *C*, $1\frac{81}{94}$ da.
71. $10\frac{22}{41}$ da.
72. 108 men.
73. 36 da.
74. 3 hr.
75. $107\frac{85}{193}$ da.
76. 8 da.
77. *A*, $6\frac{22}{27}$ da.; *B*, $9\frac{13}{19}$ da.; *C*, $14\frac{2}{13}$ da.
78. 4 da.
79. *B* walks a mile in $13\frac{1}{4}$ min.; he lost by $11\frac{1}{4}$ min., and by $\frac{45}{53}$ mi.
80. 10 points.
81. $3\frac{1}{2}$ da.
82. $3\frac{1}{3}$ hr.
83. $28\frac{4}{7}$ da.
84. $7\frac{1}{3}$ mi.
85. $16\frac{2}{3}$ mi.
86. 48 min.
87. 6 hr. 30 min. p.m.
88. $1\frac{9}{17}$ hr.
89. $7\frac{1}{5}$ hr.; 18 hr.; $5\frac{1}{8}$ hr.
90. 10 hr.
91. $3\frac{9}{13}$ da.
92. $2.8\dot{5}2330\dot{9}$ hr.
93. 32 da.
94. 9 da. of 8 hr. each.
95. 70 da.
96. 18 min.
97. *A*, $\frac{1}{3}$; *B*, $\frac{14}{45}$; *C*, $\frac{16}{45}$.
98. $5\frac{3}{7}$ da.
99. $\frac{3}{85}$ more.
101. 7 mi. 1048.3 yd.
102. 5 hr. 48 min.
103. 50 mi.; 30 mi.
104. 680 m.
105. \$.029$\frac{23}{138}$.
106. \$269.33$\frac{1}{3}$.
107. \$2408.40.
108. 300 men.
109. 324 da.
110. 3136 m.
111. 18 da.
112. 1200 men.
113. $\frac{1}{11}$ min.
114. $33\frac{1}{3}$ da.; $\frac{1}{5}$.
115. 60 min.
116. 45 mi. and 30 mi. per hour.
117. 30 mi. and 25 mi. per hour.
118. 360 da.
119. 60 mi. from *A*'s starting point; 5 hr. and 15 hr. from starting.
120. $10.9\frac{229}{576}$ a.m.
121. 5 hr. 20 min. p.m.
122. 120 m.
123. 52 da.
124. 18.055, 6.705.
125. 339.4 … m.; 169.7 … m.
126. 60.

127. 1000 m. 128. 296.6 Kg. 129. $1920.
130. $\$1714\frac{2}{7}$. 131. $\$6\frac{16}{19}$. 132. 17254.3.
133. 184 Kg. 135. 42.45 mi.
136. Men, $\$227.70$; women, $\$91.08$; children, $\$60.72$.
137. $762\frac{1}{2}$ bu. at 74c.; $237\frac{1}{2}$ bu. at 82c. 138. 84, 97, 112.
139. 17 cows; 15 hogs. 140. 12 hr. $10\frac{10}{143}$ min.
141. Silk, $\$1.15$; satin, $\$1.35$. 142. 1500 m.
143. $12\frac{8}{81}$ mi. 144. 14 min. $43\frac{3}{4}$ sec. 145. 1028.78 g.
146. 277.59 cu. in. 147. 108 mi. 148. 4.40 p.m.
149. $148\frac{41}{64}$ lb. 150. $364\frac{4}{29}$ yd. 151. $\$126$.
152. $\$22181.25$. 153. $55\frac{5}{7}$c. per hour. 154. $\$14$.
155. $13\frac{11}{153}$c. 156. 14.77 ... gal. 157. $4\frac{3}{115}$ hr.
158. $\frac{5}{22}$. 159. $\frac{1}{8}$.
160. Men, $\$79\frac{241}{738}$; women, $\$108\frac{2316}{4345}$; children, $\$93\frac{123}{4345}$.
161. $116547\frac{3}{4}$ tiles. 162. (*a*) 1764 lb.; (*b*) 28.224 cu. ft.
163. $39\frac{1}{2}$ gal. 164. 2640 lb. 165. 40 ft. by 24 ft.
166. $\$8470$. 167. 560 yd. 168. $31\frac{279}{871}$ yd.
169. 60 mi. distant.

ORAL EXERCISE. PAGE 109

1. 8 da. 2. $\$135$. 3. $\$19.50$.
4. $2\frac{2}{3}$ da. 5. 15 hr. 6. $5\frac{1}{3}$ da.
7. *A*, 16 da.; *B*, $9\frac{3}{5}$ da.; *C*. 48 da. 8. 20 and 4.
9. $4\frac{1}{2}$ and $3\frac{1}{4}$. 10. 26 yd. 11. 120 mi.
12. $\$9.40$. 13. 20 mi. 14. 3 hr. 40 min.
15. 1920 books. 16. $27\frac{1}{3}$ mi. 17. $\$2.92\frac{1}{2}$.
18. 41c. 19. 64. 20. $\$5$.
21. 27 ft. 6 in. 22. 42c. 23. $\$125$. 24. 40.

EXERCISE XXIII. PAGE 111

1. (*a*) 170; 21.25. (*b*) 5170; $738.\dot{5}7142\dot{8}$. (*c*) 168263; $56087.\dot{6}$.
(*d*) 215.7; 26.9625. (*e*) 81.639; 10.204875.
2. 92. 3. 174 lb. 4. 10.4 yr.
5. $\$6.60$ 6. 36.75c. 7. 70*s*. 5*d*.
8. $10\frac{4}{19}$ yr. 10. 155 lb. 11. 10139 ft.

12. *A*'s, $1200; *B*'s, $900.
13. 2½ doz. and 7½ doz.
14. 7 boys to 3 girls.
15. 51233.
16. 539.2.
17. $12.
18. 29.4.
19. 65.625.
20. 13.25 yr.
21. 51 lb.
22. 48.35 gr.
23. $9150; 45¾c.
24. 12.5$\dot{8}$ yr.
25. 11.368 carats.
26. 119 lb.
27. 448.6$\dot{3}$ Km.; 56.0791$\dot{6}$ Km.
28. 774.4; 1765.4; 1762.1; 1718.75.
29. 169.5 cm.
30. $12.80.
31. 51°.
32. 186.5 lb.
33. 91 Kg. 268.75 g.

Oral Exercise. Page 115

1. 245.
2. 43.75.
3. 1 lb. 1.$\dot{6}$ oz.
4. 97 lb.
5. 106.4 lb.
6. $35.
7. 6.5c.
8. 2 oz.
9. 10500 m.
10. 5.75.
11. $22.50.
12. 15 lb.
13. 32 lb.
14. 11 ft. 8 in.
15. 39 mi.; 9.75 mi.
16. 110; 22.
17. 5½ in.
18. 41.$\dot{6}$
19. 4 29/30; .99 ... in.
20. 560 pages.

Exercise XXIV. Page 117

1. (*a*) $120. (*b*) 278 horses. (*c*) $202.50. (*d*) 4 ac.
(*e*) 216 books. (*f*) 30 yd.
2. 31970.
3. 148 pupils.
4. 4500 qt.
5. $700.57.
6. 44%.
7. (*a*) 33⅓%. (*b*) 20%. (*c*) 1½%. (*d*) 40%. (*e*) 2½%. (*f*) 11¼%.
8. 20%.
9. 33.11%.
10. 3¾%; 12½%; 15%.
11. 1¾%.
12. 11⅑%.
13. 1500%.
14. 64%.
15. $280.
16. 300; 23437¼; 4300000.
17. 1875.
18. $8100.
19. 1000.
20. 200.
21. 330.
22. 360 bu.
23. 27900.
24. 60 gal.
25. $3584.
26. $823.68.
27. $4906.25.
28. 4%; 5%.
29. $450.
30. 50000000 qr.
31. $4000.
32. 750 sheep.

33. Capital, \$1000000; receipts, \$100000. 34. \$20000.
35. \$3000. 36. \$10000; \$4000. 37. £1160.
38. $14\frac{38}{63}$ yd. 39. \$2. 40. 5 : 4.
41. Barley, 40c.; wheat, 75c. 42. 277.5 dcm.
43. 214415. 44. \$1362 nearly. 45. $1764\frac{12}{17}$ ha.
46. 150; 100; 90. 47. 75 girls. 48. 19%.

ORAL EXERCISE. PAGE 122

1. $33\frac{1}{3}$%. 2. 24%. 3. 54 sheep.
4. 30; 45; 48. 5. $\frac{1}{20}$; $\frac{7}{100}$; $\frac{1}{10}$; $\frac{1}{5}$. 6. $\frac{1}{100}$; $\frac{1}{50}$; $\frac{1}{20}$; $\frac{1}{200}$.
7. \$150. 8. \$1600. 9. \$2000.
10. 27 mi. 11. 200 ac. 12. \$1380.
13. 80%. 14. 80%. 15. 60 pupils.
16. 330 pupils. 17. \$20.

EXERCISE XXV. PAGE 124

1. (*a*) \$196. (*b*) \$256.50. (*c*) \$2103.30. 2. \$248.
3. $33\frac{3}{5}$%. 4. $\$33\frac{1}{3}$. 5. $32\frac{1}{2}$%.
6. \$1.25. 7. $7\frac{9}{13}$. 8. 0%.
9. \$113.40. 10. \$16. 11. Loss, $14\frac{1}{2}$%.
12. \$8. 13. \$11. 14. $46\frac{14}{171}$%.
15. \$15.12. 16. \$450. 17. \$240.
18. $\$414\frac{22}{27}$. 19. \$320. 20. 300 shovels.
21. \$1.08 gained per gross. 22. \$13.04.
23. \$151.20. 24. \$28.50. 25. $11\frac{1}{9}$%.
26. 25%. 27. 22.54 ... %. 28. \$1.60.
29. \$413.34.

ORAL EXERCISE. PAGE 127

3. \$192. 4. \$625. 5. 28%.
6. \$625. 7. \$50. 8. \$720.
9. 20%. 10. 20%. 11. \$27.50.
12. 25%. 13. $11\frac{1}{9}$%. 14. 160.
15. 455. 16. 28%. 17. 0.
18. \$6.25. 19. \$80. 20. \$5.

Exercise XXVI. Page 128

1. 25%.
2. $45\frac{5}{11}$%.
3. 4.91 … %.
4. $33\frac{1}{3}$%.
5. 50%.
6. Gained 13.9 … %.
7. 40c.
8. $9.
9. 24 lb.
10. $3.50.
11. $3500.
12. $2000.
13. $4.35.
14 £1 1*s.* 1*d.*
15. Gained $3\frac{13}{17}$%.
16. $600.
17. $15.
18. 8%.
19. 10%.
20. $300.
21. 3*s.* $7\frac{1}{4}$*d.*
22. 4 lb.
23. $3.45.
24. $3.60.
25. $33\frac{1}{3}$%.
26. $99\frac{3}{4}$c.
27. $40.
28. $224.
29. $360.
30. $400.
31. $66\frac{2}{3}$%.
32. $180 ; $360.
33. $800.
34. 100%.
35. Loss, 40%.
36. Loses 4%.
37. $214.
38. 6.17\frac{1}{2}$.
39. $56\frac{1}{4}$%.
40. $4500.
41. Loses 40%.
42. 6c.
43. $3.40.
44. $37\frac{5}{51}$%.
45. $92 ; $115.
46. Loss, $1.
47. $200.
48. Loss, 5%.
49. $6.50.
50. $5.60.
51. $4.30.
52. 50%.
53. 8%.
54. $80 ; 133\frac{1}{3}$; loss, 13\frac{1}{3}$.
55. $6\frac{2}{3}$%.
56. $73\frac{2}{3}$c.
57. Loses $\frac{1}{4}$%.
58. 1\frac{17}{18}$; loss, $2\frac{7}{9}$%.
59. $26\frac{2}{3}$c.; loss $11\frac{1}{9}$%.
60. Loss, $200.
61. $48.
62. $1800.
63. $1600 ; $2400.
64. 80%.
65. Loss, $6\frac{1}{4}$%.
66. $1298.96 …
67. $1.

Oral Exercise. Page 135

1. 30%.
2. $4\frac{16}{21}$%.
3. 25%.
4. 40%.
5. 10%.
6. $29.
7. 12c.; 72c.
8. $175.
9. $55.
10. $1350 ; $150.
11. $33\frac{1}{3}$%.
12. 20%.
13. $14\frac{2}{7}$%.
14. $46\frac{2}{3}$%.
15. $80.
16. $20.
17. Loss, $16.
18. 60c.
19. 40c.
20. $2.20.
21. 60c.
22. 250 yd.
23. $37\frac{1}{2}$%.
24. $1.50.

EXERCISE XXVII. PAGE 138

1. (*a*) $106.40. (*b*) $700. 2. $167.37½.
3. $7488. 4. $26.66. 5. 4½%.
6. 2¼%. 7. 5⅓%. 8. 1½%.
9. $38400. 10. $7840. 11. $3750.
12. $712.50. 13. $77. 14. 600 bbl.
15. $20000; $50. 16. $9.50. 17. ⅞%.
18. $9800. 19. 2%. 20. 96⅞c.
21. 7760 yd. 22. $5141. 23. 3⅗%.
24. $7175. 25. $819; $1365; $25935.
26. 3½%. 27. $1.12½. 28. 83⅓c.
29. 1280 ac. 30. $14100. 31. $11165.
32. 2½%. 33. $388. 34. 4%; 2%.
35. 4%. 36. $1916.25. 37. $1200; $39.90.
38. 20000 lb.; $35.88.

ORAL EXERCISE. PAGE 143.

1. $8.40. 2. $12.50. 3. $540.
4. $432. 5. $15. 6. 3%.
7. 4½%. 8. $12.60. 9. 2½%.
10. $356. 11. 3%. 12. $500.
13. $780. 14. 4%. 15. 12½c.
16. $30. 17. 500 ac. 18. $1200.
19. $384.

EXERCISE XXVIII. PAGE 145

1. $3.12½. 2. $747.25. 3. £4488 15*s*.
4. 415.25. 5. $473. 6. $9.80.
7. $10000. 8. $4800. 9. $9930.
10. $53⅓. 11. $2450. 12. .6%. 13. ⅔%.
14. (*a*) $750. (*b*) $312.50. (*c*) $24562.50. (*d*) $25750.
15. Claim, $70000; loss, $5000. 16. $56.25.
17. $54. 18. $30000. 19. 1 23⁄30%.
20. $15246 2⁄13. 21. $164.44̇.

ORAL EXERCISE. PAGE 148

1. $31.70. 2. $14. 3. $24.
4. $60. 5. $12000. 6. $1900.
7. $2400. 8. $3600; $6000. 9. $\frac{3}{4}\%$.
10. $1\frac{1}{2}\%$. 11. $\frac{3}{4}\%$. 12. $\frac{1}{8}\%$.
13. $1200. 14. $1\frac{1}{4}\%$. 15. $162.50.
16. $\frac{3}{8}\%$. 17. $63. 18. $4800.

EXERCISE XXIX. PAGE 150

1. $56.70. 2. $0.017. 3. $2437500.
4. $1\frac{1}{4}$c. on the dollar. 5. $16\frac{1}{2}$ mills on the dollar.
6. $25.07. 7. $2300. 8. $63.
9. $1\frac{3}{4}$c. 10. $7800. 11. $1440.
12. $10500. 13. $4000000. 14. $1\frac{1}{3}$c.; $96.
15. $6000. 16. 12 mills on the dollar.
17. $12. 18. 1st; $2\frac{2}{3}$c.
19. 16 mills on the dollar.
20. 18 mills on the dollar. 21. $2500.
22. $275000. 23. $22.50. 24. $2400.
25. $12\frac{1}{2}$ mills on the dollar.

ORAL EXERCISE. PAGE 152

1. $29400. 2. $1600. 3. 14 mills.
4. $5700. 5. 15 mills on the dollar.
6. $3750. 7. $12000. 8. $70.
9. $67.50. 10. $14000. 11. $60.
12. $240. 13. $21.
14. 8 mills on the dollar. 15. $4000.
16. $18620.

EXERCISE XXX. PAGE 154

1. $916.20. 2. $616. 3. $218.75.
4. $262.50. 5. $4750. 6. $1179.
7. 35%. 8. $8.88. 9. $214.49.
10. $84. 11. $210. 12. 35%.

13. 300 lb. 14. $7.50. 15. 25%.
16. $270. 17. $1.50. 18. 30%.
19. $3 per pound. 20. $3402.

Oral Exercise. Page 156

1. $11.25. 2. $22.68. 3. $172.80.
4. $40. 5. 50c. per pound. 6. $1.35 per pound.
7. $38.09. 8. 27½%; ad valorem.
9. $27.60. 10. $1512. 11. 52c.
12. $300. 13. 4c. per pound. 14. $4000.
15. 25%. 16. 3c.
17. 40c. per hundredweight. 18. $1500.
19. $109.20. 20. 40%. 21. 8⅓%.
22. $1.25 per ton.

Exercise XXXI. Page 159

1. $53.58⅗. 2. $22. 3. $45.50.
4. $40.

Exercise XXXII. Page 161

1. (*a*) $63.80. (*b*) $412.50. (*c*) $870. (*d*) $729. (*e*) $3441.75. (*f*) $156. (*g*) $164.02½.
2. $4789.50. 3. $11677.20. 4. Gains $22.50.
5. 4½%. 6. 5¼ yr. 7. $1350.
8. $375. 9. 20 yr. 10. 5¼%.
11. 12½ yr. 12. 5%. 13. 9⅛%.
14. Lost $5. 15. 6%. 16. $600; 13½ yr.
17. October 6th. 18. $320. 19. $250; 6%.
20. $360; 2½ yr. 21. $1785. 22. 7%.
23. $1680. 24. 7½%. 25. 13⅓ yr.
26. $7500. 27. 5%; 10%.
28. Second is greater by $50. 29. 4%.
30. 16 yr. 32. 30 yr. 33. $1251.81.
34. $150. 35. $138.60. 36. $4800.
37. 4¼% and 8½%. 38. $1228.80. 39. $13125.
40. $349.72.

ORAL EXERCISE. PAGE 165

1. .09.
2. $94.50.
3. $105.
4. $\frac{11}{20}$.
5. $81.
6. 8%.
7. 6%.
8. 6%.
9. $1000.
10. $1\frac{1}{3}$ yr.
11. $2\frac{1}{4}$ yr.
12. $3\frac{3}{4}$ yr.
13. 25 yr.; 20 yr.; $16\frac{2}{3}$ yr.
14. 7%.
15. $8\frac{1}{3}$%; 10%; 5%.
16. $862.50.
17. $96.
18. $525.
19. $550.
20. 6%.
21. $500; 8%.
22. $7\frac{1}{2}$%.
23. $4590.
24. 3%.
25. 8%.
26. $1\frac{1}{3}$ yr.
27. 5 yr.
28. $1150.

EXERCISE XXXIII. PAGE 171

1. (*a*) Dec. 3; 108 da.; $985.20; $14.80.
   (*b*) June 24; 73 da.; $602.34; $9.79.
   (*c*) June 20; 66 da.; $501.55; $7.36.
2. (*a*) $393.34. (*b*) $1584.44. (*c*) $577.40. (*d*) $722.
3. $714.28...
4. 1333\frac{1}{3}$.
5. $485.
6. $9\frac{13}{43}$%.
7. $838.02...; $25.98.
8. $63.52.
9. $25.50.
10. $11.22...
11. $7\frac{1}{2}$%.
12. $6\frac{1}{4}$%.
13. 73 da.
14. $292.
15. $8\frac{1}{3}$%.
16. Sept. 11.
17. $800.
18. $\frac{197}{200}$.
19. $312.
20. $7\frac{1}{2}$%.
21. $2919.68.
22. $7\frac{1}{2}$%.
23. $11.53.
24. $10\frac{10}{49}$%.
25. $1666.46.
26. $1125.
27. $300.11.
28. $3046.57...

ORAL EXERCISE. PAGE 174

1. $3.
2. $241.
3. $18.
4. $250.
5. $7\frac{1}{2}$%.
6. 146 da.
7. $500.
8. $\frac{9}{200}$.
9. $\frac{4}{125}$.
10. $121; $125.
11. 70 da.
12. $500.
13. $75.
14. 69 da.

COMPOUND INTEREST. PAGE 176

1. (*a*) $59.109. (*b*) $107.733. (*c*) $248.77. (*d*) $160.32.
2. (*a*) $1141.166. (*b*) $843.648. (*c*) $1458.6075. (*d*) $1159.274.

EXERCISE XXXIV. PAGE 177

1. $125.509.
2. $238.81.
3. $1689.73 ...
4. $132.405.
5. $1157.308.
6. $153.22.
7. $5.508.
8. £4 3*s*. $2\frac{74}{125}d$.
9. $420.25.
10. 92c.
11. $15400.
12. $24000.
13. $1200.
14. 4%.
15. $1000; 4%.
16. $800; 5%.
17. $7\frac{49}{400}\%$.
18. 4.88 ... %.
19. $560.
20. $1200.
21. $5\frac{1}{2}$ yr.
22. $2500.
23. 12800.
25. (*a*) $\$2\frac{2}{3}$; (*b*) $4\frac{1}{2}\%$.
26. $3750.
27. 8.243 ... %.
28. $16000.
29. $410; $800.
30. $150000; 4%.
31. $33007.71.
32. $1810.25.
33. $10000.
34. 10648.
35. $30375.
36. $62500.
37. 8%; $625.
38. $1771.41 ...
39. $3038.765 ...
40. 250 mi.; 300 mi.; 360 mi.; 432 mi.

ORAL EXERCISE PAGE 181

1. $\frac{26}{25}$.
2. $\frac{676}{625}$.
3. $(\frac{26}{25})^3$.
4. $\$\frac{216}{100}$.
5. $1.50.
6. $\$\{750(1.05)^4\}$.
7. $\$(1.03)^4$.
8. $125.
9. $\$(1.05)^3$; $(1.05)^4$; $\$(1.05)^4$.
10. 9680.
11. 5%.
12. 25%.
13. $882.
14. $\$[400\{(1.05)^4 - 1\}]$.

REVIEW EXERCISE XXXV. PAGE 182

1. 4.065%.
2. $225.
3. $3640.
4. 41825.
5. 10%.
6. $1760.
7. $7119.80.
8. 21.75.
9. $10935000.
10. *A*, $40; *B*, $45.
11. $8400.
12. 418 bales; $323.58.
13. As 40 to 31.
14. $8.
15. 2.97*d*.

16. \$305.78$\frac{1}{3}$.
17. 16222.11 lb.
18. 165; 255; 330.
19. Grain, \$1020; groceries, \$950.
20. *A*, \$842.30; *B*, \$918.87; *C*, \$1598.83.
21. \$4500.
22. \$2040.
23. 100 bales.
24. \$255.
25. \$3000.
26. \$46$\frac{2}{3}$.
27. \$2568.
28. Loss, 8$\frac{4}{7}$%.
29. \$1.86$\frac{13}{43}$.
30. 21$\frac{1}{2}$%.
31. Loss, \$10.163 …
32. \$2.32$\frac{3}{4}$.
33. 11$\frac{1}{9}$%; 8$\frac{7}{8}$%.
34. 9$\frac{11}{16}$c.
35. 4000 lb.; \$1.08$\frac{1}{2}$.
36. \$80.
37. *A*, \$2.67$\frac{5}{7}$; \$2.68$\frac{3}{4}$; 40%; 39$\frac{23}{43}$%.
38. \$10.
39. Gain, 12.99%.
40. Lost, \$71$\frac{19}{81}$.
41. \$6000; \$8160; \$6528.
42. \$2530.
43. 633$\frac{3}{5}$ lb.
44. 1 alloy to 2 gold.
45. (*a*) \$0.548077. (*b*) \$0.899. (*c*) \$855.

Exercise XXXVI. Page 194

1. (*a*) \$8562.40. (*b*) \$9310. (*c*) \$11207.25. (*d*) 3542. (*e*) £523 16*s*. 9*d*.
2. (*a*) \$11200. (*b*) \$2700. (*c*) \$850. (*d*) £2400. (*e*) £6000.
3. (*a*) \$58.80. (*b*) \$171.60. (*c*) \$385. (*d*) \$150. (*e*) £276.
4. (*a*) \$600. (*b*) \$680. (*c*) \$550. (*d*) \$3200.
5. (*a*) 6$\frac{2}{3}$%. (*b*) 5$\frac{5}{19}$%. (*c*) 5$\frac{10}{13}$%. (*d*) 4$\frac{2}{3}$%.
6. (*a*) 8$\frac{1}{3}$%. (*b*) 7$\frac{1}{2}$%. (*c*) 7$\frac{1}{3}$%.
7. (*a*) 140. (*b*) 70. (*c*) 270.
8. (*a*) \$9600. (*b*) \$67500. (*c*) \$41540.
9. (*a*) 101$\frac{1}{2}$. (*b*) 85. (*c*) 92$\frac{1}{8}$.
10. 83$\frac{1}{3}$.
11. 6 per cents.; $\frac{4}{5}$%.
12. £4725.
13. \$450.
14. \$4984.80.
15. 223.
16. 60%.
17. £24960.
18. 90.
19. Nothing.
20. Increased; \$56.55.
21. \$10692; \$21384.
22. \$8000.
23. 5$\frac{3}{5}$ yr.
24. Loss, \$1.70 …
25. \$6000.
26. 6 per cents.
27. \$40000.
28. \$44092.
29. 89$\frac{3}{4}$.
30. \$23.
31. 4$\frac{4}{249}$%; 4$\frac{9}{34}$.
32. \$3200000.
33. \$6000; \$3000.
34. \$21500.
35. \$24044.25.
36. 4 per cents.; \$128700.
37. \$2.45.

38. $1.50.
40. $7\frac{1}{2}$%; $50 less.
41. 90; $465.
42. $136.458...; $6\frac{1}{4}$%.
43. £60000.
44. 87.5.
45. $760.
46. $2035.
47. 23.17\frac{31}{33}$.
48. $24360.
49. Increase, $59.40.
50. 15208\frac{1}{3}$.
51. $305.
52. £6000.
53. 1500 bbl.
54. Increase, £66$\frac{2}{3}$
55. Loss, $25; increase, 16\frac{2}{3}$.
56. First, by 88\frac{1}{3}$.
57. $2200.
58. $66.
59. $34947.
60. $6900.
61. 90.
62. $102.723 ...
63. $1650.
64. $1000.
65. 40 share
66. 5 per cents.; $136762.50.
67. 43$\frac{1}{3}$%.
68. Loss, $978; increase, $139.50.

ORAL EXERCISE. PAGE 203

1. $5625.
2. $8400.
3. $425.
4. $10400.
5. $2750.
6. $3500.
7. $7185.
8. $7700.
9. $4800.
10. $150.
11. 5%.
12. $3750.
13. 4%.
14. $1980.
15. $2500.
16. $25000.
17. $230.
18. $12000.
19. $4800.
20. Same.
21. $6480.

EXERCISE XXXVII. PAGE 206

1. $33; $27.
2. $250; $375; $875; $1000.
3. $3300; $2200; $1650; $1320.
4. [illegible] cwt. saltpetre; $1\frac{1}{3}$ cwt. sulphur; $1\frac{1}{6}$ cwt. charcoal.
5. 120 yd.; 160 yd.; 200 yd.
6. *A*, $240; *B*, $80; *C*, $320.
7. 28; 32; 40.
8. *A*, £102 3*s.* 9*d.*; *B*, £132 16*s.* 10$\frac{1}{2}$*d.*; *C*, £183 18*s.* 9*d.*
9. $4515; $10836; $12040; $9030.
10. *A*, $11.50; *B*, $5.75; *C*, $9.20.
11. Silver, $201\frac{9}{11}$ gr.; copper, $16\frac{4}{11}$ gr.
12. $988; $1490; $1734.
13. 120; 72; 45.
14. $7560; $5670.
15. $52.50; $35.
16. 12 da.
17. *A*, $2.49; *B*, $15.81.
18. 113; 339; 678; 791.
19. 30.

20. $57\frac{41}{87}$ ac.; $40\frac{20}{87}$ ac.; $91\frac{83}{87}$ ac.; $10\frac{30}{87}$ ac.
21. Men, \$5; women, \$3; boys, \$2.40.
22. Men, \$182.70; women, \$182.70; children, \$152.25.
23. \$6000. 24. *A*, \$700; *B*, \$2500; *C*, \$1800.
25. *A*, \$1050; *B*, \$1200; *C*, \$1250; *D*, \$1500.
26. $\frac{414}{1573}$; $\frac{36}{143}$; $\frac{69}{286}$.
27. \$175.50; \$218.40; \$252.72; \$117.00; \$149.76.
28. 1200 boys.
29. £142 12*s*. 6*d*.; £42 15*s*. 9*d*.; £14 5*s*. 3*d*.
30. 200; 189; 101. 31. \$1925; \$770; \$154.
32. Nitre, 22 lb.; charcoal, $4\frac{2}{5}$ lb.; sulphur, $3\frac{1}{3}$ lb.
33. \$8250. 34. \$10; \$18; \$15. 35. *A*, 88c.; *B*, $49\frac{1}{2}$c.
36. *A*, \$1155; *B*, \$572; *C*, \$259.50. 37. \$6.
38. *A*, \$17.50; *B*, \$52.50; *C*, \$105; *D*, \$175.
39. \$11835.75. 40. \$8400.
41. Child, \$1920.60; brother, \$960.30.
42. \$56000; \$48000; \$42000. 43. \$30.
44. \$2 per ounce. 45. \$12; \$8; \$4.80. 46. 10 men.
47. Men, \$79.20; women, \$70.40; children, \$52.80.
48. 15; 20. 49. 316.
50. Tea, 3*s*.; coffee, 2*s*.; cocoa, 1*s*. 8*d*. 51. \$1024.
52. \$740 at $4\frac{1}{2}$%; \$860 at $5\frac{1}{4}$%.
53. *A*, \$3200; *B*, \$4800; *C*, \$6000; *D*, \$7000.
54. Nitre, $15\frac{1}{5}$ cwt.; sulphur, $1\frac{9}{10}$ cwt.; charcoal, $2\frac{9}{10}$ cwt.
55. \$39.95. 56. Gold, 4 dwt. $22\frac{1}{4}$ gr.; alloy, $10\frac{3}{4}$ gr.
57. $3\frac{1}{2}$*d*.; $\frac{1}{2}$*d*.
58. *A*, 3240; *B*, 2196; *C*, 1944; *D*, 2052 ; *E*, 1728; 6480.
59. Man, £4 4*s*.; woman, £3; child, £1 16*s*.
60. Potash, 375 gr.; soda, 390 gr.

ORAL EXERCISE. PAGE 213

1. \$35; \$50. 2. \$40; \$60.
3. 30 marbles; 90 marbles.
4. Horse, \$200; buggy, \$175.
5. 15 apples; 20 apples; 25 apples. 6. 160; 200.
7. 2473; 2265. 8. Oats, 98 bu.; peas, 245 bu.

9. *A*, $200; *B*, $80; *C*, $60. 10. $261; $540.
11. Men, $117; women, $104.
12. *A*, $100; *B*, $200; *C*, $600.
13. $18.75; $22.50; $33.75. 14. 415 yd.; 391 yd.
15. 68; 84. 16. *A*, $150; *B*, $75; *C*, $225.
17. $9; $11; $14. 18. $110; $165; $220.
19. 40 geese; 20 turkeys.
20. Horse, $150; cow, $50. 21. $425; $375; $275.

EXERCISE XXXVIII. PAGE 216

1. First, $44.25; second, $88.50.
2. *A*, 75\frac{177}{205}$; *B*, 84\frac{12}{41}$; *C*, 101\frac{31}{205}$; *D*, 170\frac{142}{205}$.
3. *A*, $4.50; *B*, $6.75; *C*, $11.25.
4. *A*, $2062.40; *B*, $2320.20; *C*, $773.40.
5. *A*, $1350 ; *B*, $1890 ; *C*, $2100 ; *D*, $3450 ; *E*, $247.50 ; *F*, $412.50.
6. *A*'s, $11600; *B*'s, $9062.50.
7. *A*, 656\frac{820}{1649}$; *B*, 286\frac{829}{1649}$. 8. *D*, $20; *E*, $50.
9. *A*, $87.50; *B*, $120; *C*, $202.50. 10. $30; $48; $28.
11. $6084.
12. *A*, $234; *B*, $266.40; *C*, $306; *D*, $345.60.
13. $300; $200; $500. 14. $1600; 15 mo.
15. $15.30; $14.25. 16. *A*, $245; *B*, $225.
17. Johnston, $585; Wilson, $487.50; Miller, $175.50.
18. *A*, $3355; *B*, $5830; *C*, $1400.
19. *A*, $34.30; *B*, $53.90.
20. *A*, $735; *B*, $490; *C*, $367.50; *D*, $294.
21. *A*'s, 16$\frac{5}{7}$ gal., and *B*'s, 25$\frac{1}{14}$ gal.
22. *A*, $118.30; *B*, $55.90; *C*, $13.
23. *B*, $2250; *C*, $5625.

ORAL EXERCISE. PAGE 219

1. $300; $450; $600. 2. $1250.
3. $35; $45. 4. 17\frac{1}{7}$; 17\frac{1}{7}$; 25\frac{5}{7}$.
5. *B*, $160; *C*, $200; *D*, $280. 6. *A*, $180; *B*, $200.

7. *A*, $1200; *B*, $1600; *C*, $2000.
8. *A*, $4; *B*, $3; *C*, $10; *D*, $7. 9. $\$3\frac{1}{2}$; $\$4\frac{1}{2}$.
10. $2500. 11. *A*, $700; *B*, $300.
12. *A*, $500; *B*, $800. 13. 4 mo.
14. *A*, $480; *B*, $1200.

EXERCISE XXXIX. PAGE 225

1. $250.625. 2. $3613.50. 3. $405.
4. $2015. 5. $2298.05. 6. $2725.50 nearly.
7. $109\frac{1}{2}$. 8. $44693.20. 9. 1% discount.
10. $2381.145 … 11. $2483.484 … 12. £179.
13. £576 12*s.* 6*d.* 14. £1 = $4.8665. 15. 2.341% discount.
16. $2666.65. 17. $1\frac{1}{8}$% premium. 18. 2 fr. 13 cent.
19. 1760 copeks. 20. 9 fl. 20 krs. 21. .875% discount.
22. $\$238\frac{2}{21}$. 23. $7788. 24. $4500.
25. 4 marks = 95c. 26. $2559.30 … 27. 10785.3 … marks.
28. $1 = .848 … milreis. 29. $3273.55.
30. $4.86. 31. $442. 32. £52 4*s.* $6\frac{6}{61}$*d.*
33. $94\frac{1}{2}$c. for 4 marks. 34. $4.81 nearly.
35. $875. 36. $2489.72 … 37. 2% premium.
38. 5.2 fr. = $1. 39. Through London.

MISCELLANEOUS EXERCISE XL. PAGE 229

1. $78\frac{3}{4}$c.; $66\frac{3}{4}$c. 2. *A*, $4912; *B*, $6168.
3. $\$1 = 5\frac{5}{23}$ fr. 4. 10 gal. and 4 gal.
5. $33\frac{1}{3}$ lb. of 8c., 10c. and 12c., and 100 lb. of 20c.
6. $1212. 7. $\$1257\frac{1}{7}$. 8. $\frac{81}{256}$.
9. $\$2211\frac{9}{11}$. 10. $43.63.
11. Paris, $\$14285.71\frac{3}{7}$; London, $14600; Amsterdam, $14640.
12. 1 lb. at 8c.; $8\frac{1}{2}$ lb. at 13c.; $2\frac{2}{3}$ lb. at 14c.
13. $9.70. 14. 30 gal. 15. $118\frac{4}{7}$%.
16. 8, 10, and 12 mo. 17. $£9176\frac{8}{17}$. 18. .42; $23\frac{9}{17}$
19. 1.2372. 20. $5774.43.
21. Direct, $14224.91; circular, $14476.72; gain, $251.81.
22. $1 = 906.5 reis. 23. 9 spirits to 31 water.

24. 4 at 80; 8 at 85; 8 at 100; 80 at 115. 25. 13 lb.
26. 140 ac. 27. 30 at \$4; 135 at \$4$\frac{1}{2}$; 75 at \$5$\frac{1}{2}$; 30 at 7$\frac{1}{2}$
28. Turkey, \$1.75; goose, 85c.
29. 360 bu. at 65c; 640 bu. at 68c. 30. 25 gal.
31. 35c. 32. 2 water to 10 milk.
33. 12 gal. 34. 2$\frac{17}{24}$ oz. 35. £2 3*s.* 2$\frac{1}{2}$*d.* nearly.

Exercise XLI. Page 234

1. $\frac{4}{9}$ is greater. 2. $\frac{19}{41}$ is greater.
3. $\frac{8}{15}$ is least; $\frac{4}{7}$ is greatest. 4. 45 : 364.
5. 112 : 405. 6. \$31.25. 8. 25 : 36.
9. $\frac{13}{40}$. 10. $\frac{1}{2}$; 3$\frac{1}{3}$. 11. 128 : 1.
12. 9 : 13. 14. *A*'s rate; *B*'s as 159 : 120.
15. 2 hr. 36 min. p.m. 16. 63.66 … ft.
17. 49.49 in. 18. 40. 19. 4$\frac{5}{6}$.

Exercise XLII. Page 237

1. 4 : 3 :: 12 : 9. 2. 12$\frac{3}{4}$. 3. .0076.
4. *A* : *C* :: 25 : 39. 5. 21. 6. $\frac{10}{21}$.
7. .048. 8. 28. 9. $\frac{5}{27}$.
10. 17.68. 11. \$1560. 12. $\frac{48}{175}$.
13. *A*, \$552; *B*, \$460; *C*, \$345; *D*, \$230.
14. *A*, \$3000; *B*, \$4500; *C*, \$5625; *D*, \$6562.50.
15. *A*, \$67.50; *B*, \$54; *C*, \$45. 16. 55, 77, 99, 121.
17. \$12000. 18. \$16$\frac{7}{8}$. 19. \$47.13$\frac{1}{3}$.
20. 3$\frac{39}{112}$ ft. 21. 78$\frac{3}{4}$ yd. 22. \$902$\frac{7}{9}$.
23. \$29.25. 24. \$21.59. 25. 4 men.
26. 7722 stones. 27. 12800 28. 6$\frac{28}{37}$ hr.
29. 2025 bu. 30. 28$\frac{1}{3}$ da. 31. 10 da.
32. £1285. 33. 10 hr. 40 min. 36$\frac{7}{18}$ sec.
34. $\frac{6}{13}$ mi. 35. 3 hr. 25 min. p.m.
36. 10 da.; 12$\frac{6}{7}$ da. 37. 427$\frac{5}{6}$ mi.

Oral Exercise. Page 240

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{7}$. 2. $\frac{5}{6}$; $\frac{5}{4}$; $\frac{4}{9}$. 3. $\frac{3}{5}$.
4. 3 : 4. 5. 15 : 16. 6. 18.

7. 36.
8. 32 mm.
9. \$30.
10. 15 hr.
11. 60c.
12. \$40.
13. $\$12\frac{1}{2}$.
14. 340 fr.
15. 4.
16. $46\frac{7}{8}$ fr.
17. 15 oranges.
18. 6750 Kg.
19. 225 ft.
20. $25\frac{5}{7}$ da.
21. 108 revolutions.
22. $\$65\frac{1}{3}$.
23. \$84.
24. \$220.
25. 4.56 g.
26. \$84.
27. \$1212.80.

EXERCISE XLIII. PAGE 244

10. Rate, 8 mi.; 8 mi. from starting point.
16. (*b*) 27.1 in.; 28.3 in.; 22.3 in.
18. (*b*) 12.5° C.; 12° R.
23. (*c*) 1876, 33.3 millions; 1895, 39.7 millions.

EXERCISE XLIV. PAGE 251

1. (*a*) $300\frac{3}{8}$ sq. ft. (*b*) 12 sq. ft. (*c*) 3.894 sq. m. (*d*) $452\frac{5}{6}$ sq. ft. (*e*) $356\frac{1}{2}$ sq. ft. (*f*) 4.512 sq. dm.
2. (*a*) $1406\frac{1}{4}$ sq. yd. (*b*) $315\frac{1}{16}$ sq. ft. (*c*) 12.96 sq. m. (*d*) 91 sq. ft. 121 sq. in. (*e*) 2232 sq. ft. 81 sq in. (*f*) 34.81 sq. dm.
3. (*a*) 8 ft. 9 in. (*b*) 11 yd. (*c*) 88 yd. (*d*) 99 yd. (*e*) 60 m.
4. (*a*) 103 ft. (*b*) 405 ft.
5. 64 sq. in.

EXERCISE XLV. PAGE 252

1. $86\frac{2}{3}$ sq. yd.
2. \$8712.
3. $\$35.46\frac{2}{3}$.
4. $\$106.66\frac{2}{3}$.
5. \$48.
6. \$2.40 …
7. \$69.42 …
8. 18.5 m.
9. (*a*) 500 sq. ft.; 100 ft.
10. (*a*) $4\frac{3}{4}$ mi. (*b*) 610 ac.
11. $\$50.93\frac{1}{3}$.

EXERCISE XLVI. PAGE 254

1. 80 sq. yd.
2. $310\frac{2}{3}$ sq. yd.
3. $\$56.87\frac{1}{2}$.
4. 6.109 … bunches.
5. \$54.
6. \$184.80.

Ceiling not
in [illegible]

EXERCISE XLVII. PAGE 256

1. (*a*) $30\frac{1}{3}$ yd. (*b*) $89\frac{1}{3}$ yd. nearly. (*c*) $145\frac{1}{3}$ yd.
(*d*) $45\frac{5}{8}$ yd. nearly.
2. (*a*) \$33.60. (*b*) \$93. (*c*) \$84.60. (*d*) £11 18*s.* 6*d.*
(*e*) \$161.60. (*f*) 1531.16 ... fr.; \$296.02.
3. (*a*) $\$57\frac{5}{18}$. (*b*) \$61.215. (*c*) \$176.85

EXERCISE XLVIII. PAGE 257

1. (*a*) $\$22.7\dot{5}$. (*b*) \$12.97... (*c*) £6 13*s.* $2\frac{1}{3}d$. (*d*) £6 0*s.* $9\frac{3}{7}d$.
2. $\$18.37\frac{1}{2}$. 3. $\$22.19\frac{1}{2}$.

EXERCISE XLIX. PAGE 258

1. (*a*) 28 sq. per. (*b*) $21\frac{1}{12}$ sq. yd. (*c*) $252\frac{15}{16}$ sq. ft.
(*d*) 264.74 sq. m. 2. $137\frac{1}{2}$ yd.
3. $2\frac{1}{2}$ ac. 4. 1120 yd. 5. 55 ft.

EXERCISE L. PAGE 259

1. (*a*) 105 sq. ft. (*b*) $688\frac{1}{2}$ sq. in. (*c*) 54.192 ac.
(*d*) 926.00625 sq. m. 2. 27 ft.
3. $27\frac{13}{15}$ chains. 4. $237\frac{1}{3}$ ft. 5. 3.8 m.

EXERCISE LI. PAGE 261

1. 19500 sq. ft. 2. 1.875 ac. 3. 151.5285 ares.
4. 36 ft. 5. $28\frac{2}{3}$ sq. ft. 6. 8.284 sq. ch.
7. 24.6 m.

EXERCISE LII. PAGE 263

1. (*a*) 58 ft. (*b*) 169 in. (*c*) 185 ft. (*d*) 274 yd.
2. (*a*) 374 in. (*b*) 608 in. (*c*) 1012 yd. (*d*) 532 in.
3. (*a*) 64 yd. (*b*) 69 ft. (*c*) 503.813 ... in. (*d*) 896 ft.
4. 12 in. 5. 346 yd. 6. 17 ft.
7. \$112.80. 8. 163.2 ... yd. 9. 883.875 ft.
10. 76.12 ft. 11. 13 ft. from ground.
12. [illegible]1.6 ft. 13. 92.95 ft.

EXERCISE LIII. PAGE 26

1. (*a*) $2\frac{2}{5}$ in. (*b*) $4\frac{8}{13}$ in. (*c*) 6.72 cm. (*d*) $8\frac{32}{41}$ in.
(*e*) 10.819 ... dm.

EXERCISE LIV. PAGE 267

1. (*a*) 1026 sq. yd. (*b*) 600 sq. ft. (*c*) 2250 sq. yd.
(*d*) 84 sq. chains. (*e*) 690 sq. m. (*f*) 19536 sq. chains.
2. 14160 sq. ft.
3. On 14, 12 ft.; on 13, $12\frac{12}{13}$ ft.; on 15, $11\frac{1}{5}$ ft.
4. 133837 sq. ft. 5. 29 ft., 52 ft., 69 ft.

EXERCISE LV. PAGE 268

1. (*a*) 44 ft. (*b*) 19.8 ft. (*c*) $24866\frac{2}{7}$ mi. (*d*) 1518 ft.
2. (*a*) $59\frac{1}{2}$ ft. (*b*) 21.7 ft. (*c*) 49 cm. (*d*) 3 ft. $9\frac{1}{2}$ in.
(*e*) 8.05 mi. (*f*) 262.5 cm.
3. (*a*) 11 ft. (*b*) $102\frac{2}{3}$ ft. (*c*) 29.92 chains. (*d*) $146\frac{2}{3}$ ft.
4. 11 in.; $13\frac{1}{5}$ in.; $17\frac{3}{5}$ in.; $82\frac{1}{2}$ in. 5. $6187\frac{1}{2}$ sq. ft.
6. $4545\frac{5}{11}$ times. 7. 4136 ft. 8. 9 mi.
9. 7 ft.

EXERCISE LVI. PAGE 270

1. (*a*) 1386 sq. ft. (*b*) 3850 sq. cm. (*c*) 7546 sq. in.
(*d*) $471\frac{5}{8}$ sq. m. (*e*) $9735314\frac{2}{7}$ sq. yd. (*f*) $38028\frac{4}{7}$ sq. dm.
2. (*a*) 63 ft. (*b*) $3\frac{1}{2}$ yd. (*c*) 5.6 m. (*d*) 21 chains.
3. 110 sq. in. 4. 246.67 yd. 5. 336 cm.
6. 2464 sq. in. 7. 19.79 in.
8. 14 yd.; 11704 sq. yd.; 49896 sq. yd. 9. $962\frac{1}{2}$ sq. in.
10. $3\frac{1}{2}$ ft. 11. $282\frac{6}{7}$ in. 12. 42 ft.
13. 35.456 ... m. 14. 702.048 in.
15. Square, 3748096 sq. in.; circle, 4770304 sq. in.

EXERCISE LVII. PAGE 272

1. 36 sq. ft. 2. $5\frac{5}{12}$ sq. ft. 3. $30\frac{4}{5}$ sq. ft.
4. 8 ft. 5. Length, $42\frac{1}{6}$ in.; area, $442\frac{3}{4}$ sq. in.
6. 144°; 88 ft. 7. 118.79 dm.

Exercise LVIII. Page 273

1. 24 in. by 32 in.
2. 1 to 660.
3. 36 min.
4. $4\frac{4}{9}$ ac.
5. $1240\frac{1}{3}$ sq. ft.
6. 1280 sq. mi.
7. 250 cm.
8. $67\frac{1}{2}$ sq. ft.
9. $1\frac{9}{16}$ ac.
10. 3[illegible]$\frac{2}{5}$ ft ; $28\frac{4}{5}$ ft.
11. 220 yd.
12. $390.
13. 16 min.; $26\frac{2}{3}$ min.

Miscellaneous Exercise LIX. Page 275

1. 210 ft.
2. £13 10*s.*
3. $5.95.
4. $1.20.
5. 12 ft.
6. $17\frac{1}{2}$ ft.
7. 1 ft. $9\frac{1}{3}$ in.
8. $12.
9. 5952 stones.
10. $13.50.
11. 429 yd.; 715 yd.
12. $10511\frac{1}{9}$ sq. yd.; $2955\frac{5}{9}$ sq. yd.
13. 26 yd.
14. $69.20.
15. 2 ft.
16. 300.
17. 1125 ac.
18. 221 ft.
19. 5.6 ft.
20. 39 ft., 60 ft., 63 ft.
21. 22 ft.
22. $75\frac{1}{6}$ yd.
23. $$3346.87\frac{1}{2}$.
24. 11 sq. ft.
25. £4 1*s.* $6\frac{23}{27}$*d.*
26. $$60.75$; $$20.42\frac{1}{2}$.
27. $1238.70.
28. $70\frac{2}{3}$ yd.
29. 104 yd.
30. 23.95 ft.
31. $11\frac{1}{4}$ ft.
32. 2214 in.; 243602 sq. in.
33. 230.94 sq. ft.
34. Decrease, 2.25 sq. yd.
35. 225 sq. yd.
36. 1400 sq. yd.
37. 220 yd.
38. $187\frac{1}{2}$ sq. in.
39. 498.83 sq. in.
40. 4056 sq. ft.
41. 32 in.
42. 2.8873 ft.
43. 7854 sq. yd.
44. $113\frac{1}{3}$ ft.
45. 504 sq. ft.; 88 ft.
46. Circle, 616 sq. in.; square, 484 sq. in.; triangle, 372.58 sq. in.
47. 88 in.

Oral Exercise. Page 280

1. 1849 sq. ft.
2. 1386 sq. in.
3. 1[illegible] in.
4. 391 sq. in.
5. $25\frac{1}{2}$ yd.
6. 616 sq. dm.
7. 2250 sq. dm.
8. 50.5 m.
9. 837 board ft.
10. 26 ft.
11. $14\frac{2}{3}$ sq. ft.
12. $$2.66\frac{2}{3}$.
13. 132 in.
14. 180 times.
15. 40 yd.

16. 37 yd. 17. \$50. 18. \$$46\frac{2}{3}$.
19. 20 sq. ft. 20. 290 cu. ft. 21. 26 sq. dm.
22. $10\frac{3}{7}$ ft.; $7\frac{5}{7}$ ft.

EXERCISE LX. PAGE 283

1. (*a*) 336 cu. ft. (*b*) $83\frac{71}{96}$. (*c*) $1058\frac{17}{432}$ cu. ft. (*d*) 281.2 cu. dm.
2. $421\frac{7}{8}$ cu. in. 3. $61\frac{3}{4}$ sq. ft.; $31\frac{7}{8}$ cu. ft.
4. 8 in.; 512 cu. in. 5. $5\frac{1}{4}$ ft. 6. 9600.
7. $14088\frac{15}{16}$ t. 8. 500 men. 9. 1 ft. 7 in.
10. $2031\frac{1}{4}$ lb. 11. $\frac{1}{8}$ cwt. 12. 160.
13. $3\frac{3}{4}$ ft. 14. £38 19*s.* 2*d.*

EXERCISE LXI. PAGE 286

1. (*a*) $14\frac{2}{3}$ sq. ft. (*b*) $93\frac{1}{2}$ sq. ft. (*c*) 275 sq. m. (*d*) $4\frac{7}{12}$ sq. ft.
(*e*) 33 sq. ft. (*f*) 21.12 sq. m.
2. (*a*) $945\frac{5}{16}$ sq. in. (*b*) 163.68 sq. m. (*c*) $356\frac{8}{9}$ sq. in.
(*d*) 438.52 … sq. m.
3. $1018\frac{2}{11}$ sq. in. 4. \$$28.87\frac{1}{2}$.
5. (*a*) 176 cu. ft. (*b*) $57\frac{3}{4}$ cu. ft. (*c*) 1122.66 cu. m.
(*d*) $269\frac{1}{2}$ cu. in. (*e*) $3118\frac{1}{2}$ cu. in. (*f*) 818.60625 cu. m.
6. $19\frac{1}{4}$ cu. yd. 7. \$247.50. 8. 1188 cu. in.
9. $14\frac{2}{3}$ cu. ft. 10. $27\frac{1}{2}$ cu. ft. 11. $36\frac{2}{3}$ cu. ft.
12. $3\frac{3}{7}$ ft. per hour. 13. 14.4 in. 14. 11.2 in. nearly.
15. 1694 coins. 16. $1194\frac{2}{7}$ cu. m. 17. 3 ft.
18. 4752 lb.

EXERCISE LXII. PAGE 288

1. 1125 sq. ft. 2. 1053 sq. ft. 3. 432 sq. ft.
4. $37\frac{1}{2}$ sq. ft. 5. 12480 sq. dm.

EXERCISE LXIII. PAGE 289

1. 630 cu. ft. 2. 2250 cu. ft. 3. $342\frac{5}{8}$ cu. ft.
4. 13400 lb. 5. $3466145\frac{31}{81}$ cu. yd. 6. 9680 cu. ft.
7. 34 cu. m.

Exercise LXIV. Page 291

1. 66 sq. yd. 2. $554\frac{2}{5}$ sq. ft.; $592\frac{9}{10}$ sq. ft.
3. 242 sq. ft. 4. $6\frac{2}{3}$c. 5. 306.144 ... sq. ft.
6. $2\frac{5}{36}$ ft. 7. $22\frac{44}{63}$ sq. m. 8. $51\frac{1}{3}$ sq. ft.

Exercise LXV. Page 292

1. $1095\frac{86}{147}$ cu. ft. 2. $19\frac{1}{4}$ cu. ft. 3. 14 ft.
4. $26\frac{22}{25}$ times. 5. $55\frac{605}{864}$ cu. ft. 6. 277.2 cu. m.

Exercise LXVI. Page 294

1. (*a*) $273\frac{7}{9}$ sq. in. (*b*) 3850 sq. in. (*c*) $707\frac{1}{7}$ sq. m.
(*d*) 1386 sq. in. (*e*) 5544 sq. in. (*f*) 24.64 sq. m.
2. 14 in. 3. 21 in. 4. \$$5.47\frac{5}{9}$.
5. 46.28 ... ft. 6. $1967420052\frac{4}{7}$ sq. mi.
7. $16\frac{1}{2}$ ft. 8. $666\frac{2}{3}$ mi.

Exercise LXVII. Page 295

1. (*a*) $113\frac{1}{7}$ cu. in. (*b*) $905\frac{1}{7}$ cu. in. (*c*) 4.851 cu. m.
(*d*) $1767\frac{6}{7}$ cu. in. (*e*) $1437\frac{1}{3}$ cu. in. (*f*) $11498\frac{2}{3}$ cu. cm.
2. $822\frac{6}{7}$ cu. in. 3. $179\frac{2}{3}$ cu. ft. 4. 64000.
5. $977\frac{5}{11}$ yd. 6. 4 in. 7. .805 ... : 1.
8. 1.3817 : 1. 9. $510\frac{5}{7}$ oz. 10. $606\frac{3}{8}$ cu. in.
11. .8284 ... in. 12. 3 ft. 13. $4\frac{1}{2}$ in.

Exercise LXVIII. Page 298

1. $119\frac{1}{4}$ sq. ft. 2. $247\frac{1}{2}$ sq. ft. 3. $3300062\frac{1}{2}$ gal.
4. \$23.24 ... 5. $365\frac{365}{384}$ cu. m. 6. 225500 gal.
7. $28\frac{1}{2}$ cu. ft. 8. 88 cu. ft.

Exercise LXIX. Page 300

1. 243 oz. 2. 54 oz. 3. $5 : \sqrt{7}$.
4. 28.845 ft. 5. 64 : 125. 6. $3\frac{3}{4}$ in.
7. 13 ft. 8. .7937 ft. 9. 12 in.
10. \$10. 11. 13.39 dm. 12. $\frac{169}{343}$. 13. 7 in.

EXERCISE LXX. PAGE 301

1. 42; 33; 22.
2. $4.40.
3. 41.568 cu. ft.
4. 60 in.
5. $523\frac{3}{7}$ cu. in.
6. 35 in.
7. $20\frac{5}{8}$ in.
8. $6647\frac{2}{3}$ cu. in.
9. $269\frac{1}{2}$ cu. ft.
10. $184\frac{5}{6}$ in.
11. $45\frac{5}{6}$ cu. ft.
12. 584 lb.
13. 12 in.
14. 1828000 cu. ft.
15. $69\frac{1}{7}$ in.
16. $1571\frac{3}{7}$ sq. in.
17. $7\frac{1}{2}$ ft.; $2\frac{1}{2}$ ft.
18. $48\frac{1}{8}$ in.
19. $3823.263\dot{8}$ lb.
20. 3 ft.
21. $16\frac{1}{2}$ ft.
22. 1*s.* $1\frac{5}{9}$*d.*
23. $1.12.
24. 1200 gal.
25. 38 barges.
26. £44 13*s.* 3*d*; $\frac{1}{6}$ ft.
27. $47\frac{1}{15}$ in.
28. $81.12; 7 yd.
29. 27 in.
30. 474 pictures.
31. 32 ft.; 16 ft.; 8 ft.
32. 48000 bricks.
33. 27 ft.; 18 ft.; 12 ft.
34. 1269.74 ... sq. ft.
35. $6\sqrt[3]{3}$; $5\sqrt[3]{3}$; $4\sqrt[3]{3}$.
36. $28\frac{1}{8}$ ft.; 25 ft.
37. 391 ft.
38. 12637 ft.; 12012 ft.
39. 2.1 ac.
40. 11250 sq. yd.
41. 12 sq. chains.
42. 2016 sq. in.
43. $67\frac{1}{2}$ in.; 54 in.
44. 13 ft.
45. 5540 sq. ft.
46. 264 yd.; 165 yd.
47. $763\frac{5}{7}$ l.
48. $\frac{1}{56448}$ in.
49. $28\frac{1}{2}$; $427\frac{1}{2}$ lb.
50. $12.31.
51. 26 yd.
52. 8*s.* $2\frac{1}{3}$*d.*
53. $20.95.
54. $2\frac{1}{2}$ in.
55. 256.
56. $168\frac{3}{4}$ gal.
57. $11\frac{1}{9}$ times.
58. $15\frac{5}{8}$ times.
59. $1\frac{1}{3}$ in.
60. 216; 64; $23\frac{17}{21}$ gal.
61. $3017142\frac{6}{7}$.
62. 4 : 5.
63. 924 sq. in.
64. 40 mi.
65. 330 mi.
66. 83.136 rods.
67. $\frac{91}{216}$.
68. 98.521 ... cu. ft.
69. 15.874 in. from the apex.
70. 7.951 in.; 9.410 in.; 12.192 in.; 24.446 in.

ORAL EXERCISE. PAGE 309

1. $78\frac{3}{4}$ cu. ft.
2. $140.
3. $64.80.
4. 5 yd. by 4 yd.
5. 36 cu. ft.
6. 1925 gal.
7. 330 sq. ft.
8. 616 cu. in.
9. 110 sq. ft.
10. 11520 cubes.
11. 7920 cu. in.
12. 358 sq. in.
13. 3080 cu. in.
14. 34560 bricks.
15. $90.
16. 66 sq. m.
17. 9 cords.
18. 2550 st.
19. 8400 l.
20. $78\frac{4}{7}$ cu. in.
21. 15 ft.
22. 5 dm.
23. 8 : 9.
24. 764000 sq. ft.

MISCELLANEOUS PROBLEMS. PAGE 311

1. $\frac{488}{637}$. 2. Horse, 26.4; train, 57.
3. $\frac{10}{21}$ da. 4. Gain, 66.74%. 5. \$2.06$\frac{1}{4}$.
6. 1 hr. 4 min. 56$\frac{2}{3}$ sec.
7. .04; .05; .085; .0475; .1; .0075; .00375.
8. 240%; 7$\frac{1}{8}$%. 9. 20$\frac{1}{4}$ ft. 10. 20%.
11. 20 yr. 12. 40$\frac{10}{11}$. 13. \$1805.44 …
14. 4 da. 15. £364 11*s.* 8*d.*
16. 16 hr. 16 min. 34$\frac{14}{15}$ sec. 17. 28957$\frac{1}{2}$ bricks.
18. 225; 5.4. 19. 7$\frac{1}{2}$%.
20. \$565.60; \$484.80; \$636.30. 21. 7 ft.
22. \$157.03. 23. 14$\frac{2}{7}$%. 24. 334.4 … %.
25. \$400. 26. 17. 27. 1$\frac{10}{11}$ ft.
28. 7$\frac{13}{16}$. 30. 216 ft.; 162 ft. 31. \$2778.30; 10$\frac{61}{120}$%.
32. $\frac{90145}{309622}$. 33. 37$\frac{1}{2}$%.
34. Monday; 4 hr. 3 min. 12$\frac{2}{3}$ sec. p.m.
35. Sphere by 1741$\frac{26}{81}$ cu. in. 36. 4 men.
37. 3$\frac{9}{16}$c. 38. \$1875. 39 1275 bbl.
40. 2$\frac{5}{8}$ ac. 41. \$340.60 42. \$2236.58 …
43. \$120. 44. \$1560.96. 45. 120; 119$\frac{7}{8}$.
46. \$730.91 … 47. *A*'s, \$150; *B*'s, \$160.
48. \$0.489 … 49. \$5120. 50. \$90.89 …
51. 21.213 … in. 52. 23$\frac{11}{43}$%. 53. 68$\frac{4}{9}$ yd.
54. 1$\frac{607}{1400}$. 55. 130.86 … gal. 56. 15 bu.
57. No. 58. 34$\frac{43}{162}$. 59. \$10200.
60. \$730. 61. 3.1748 in. 62. 36.33 l.
63. \$8. 64. 12$\frac{60}{79}$ da. 65. 24 ft.
66. \$94$\frac{2}{3}$. 67. 16 sq. ft. 68. \$7480$\frac{10}{13}$ …
69. 5%; 6$\frac{1}{2}$%. 70. \$2880. 71. \$1.50.
72. 66$\frac{11}{14}$ sq. ft. 73. Cube, 24 sq. ft.; sphere, 19.3465 sq. ft.
74. \$47 30. 75. \$9000. 76. 1 gr. nearly.
77. \$2500. 78. \$3404.25. 79. .0178285714.
80. \$400. 81. *A*, \$122; *B*, \$147.
82. 24%. 83. \$37.50. 84. \$274.50.
85. \$1750. 86. 37$\frac{251}{847}$ min. 87. \$5376.

88. 4320 Kg. 89. $20\frac{20}{291}$. 90. 643.761957 ... l
91. 150 ac. 92. $5445. 93. 16.106 ... mi.
94. 78 lb. 95. 32 da. 96. *A*'s : *B*'s as 20 : 19.
97. 40c. 98. $14.28. 99. 5%; $2.05.
100. .106.408 ... cu. in.; $137\frac{1}{2}$ sq. in. 101. $9\frac{69}{280}$
102. *A*, $\$2\frac{21}{32}$; *B*, $\$2\frac{11}{32}$; *C*, $\$\frac{25}{32}$. 103. $1275.
104. $\$1934\frac{22}{67}$. 105. $4899.67. 106. $998.40.
107. 1018.76 ... 108. 1600 l. 109. $1209.50.
110. $146\frac{2}{3}$ cu. in.; 168.7 ... sq. in. 111. $10312.50.
112. $9\frac{3}{8}$ da. 113. $1800.
114. $420 at $5\frac{1}{2}$%; $580 at 6%. 115. $4000.
116. $64\frac{2}{3}$ ft. 117. $85\frac{1}{12}$. 118. $5820.
119. 40 persons. 120. $15.12.
121. $326 at 6%; $400 at 8%. 122. 234 doz.
123. 10.187375. 124. (*b*) £6 16*s.* 5*d.* 125. 4528 sq. yd.
126. $3\frac{53}{63}$ in. 127. 2 gal.
128. 8th, 15; 9th, 25; 10th, 35. 129. 1541.
130. $37\frac{1}{2}$ yd. 131. $4.\dot{6}0\dot{5}$. 132. 75 ft.; 45 ft.
133. $2501.52. 134. $168. 135. $5300.
136. 4000 bu. 137. 39250 ft. 138. 12 ft.
139. 2 mistakes.
140. 7.241917 km. per hour; 120.6986 m. per minute.
141. 91 ac. 1 rd. 5 sq. ft. 36 sq. in. 142. $154\frac{11}{16}$ t.
143. 20% premium. 144. 50%. 145. $261\frac{1}{3}$ cu. ft.
146. 25 ft. 147. 5 mo. 148. Gain, $6.75.
149. $273. 150. *A* gets $4500; *B*, $4375; *C*, $4725.
151. $800. 152. 14.77234 gal. 153. 27.141%.
154. $730. 155. 37225. 156. $14\frac{17}{22}$% discount
157. 6 da. 158. $\$1140\frac{5}{8}$. 159. $3330.
160. $4\frac{7}{162}$ min. 161. 277 roubles 55 copeks.
162. 12° 27′ E. long. 163. $32.40. 164. 1%.
166. 2704 yd. 167. $606.28. 168. 820 bu.
169. 15 votes. 170. 3 ar. 171. $\$39.94\frac{4}{5}$.
172. $16\frac{1}{5}$ ac. 173. $75\sqrt{3}$ sq. ft. 174. $306.46.
175. 2 hr. 176. $4000. 177. $900.
178. $137\frac{1}{2}$ sq. ft. 179. Increased $99.54.

180. 200. 181. 20 ft.; 15 ft.; 10 ft.
182. $5000; $8750; $11250. 183. $12\frac{1}{2}$%.
184. 48 men. 186. 6 hr. 25 min. 52 sec. p.m., Wednesday.
187. 4% and 5%. 188. 31.
189. Surface, 3828 sq. ft.; volume, 12320 cu. ft.
190. *A*, = $675; *B*, = $600; *C*, = $500. 191. $400.
192. $$1714\frac{3}{7}$. 193. $250. 194. $25.
195. $33\frac{1}{3}$%; 600%. 196. 3325.44 Kg. 197. $\frac{1}{4}$ yd.
198. Each woman gets 70c. and each girl 40c.
199. 20.5. 200. $50. 201. $16107\frac{1}{7}$ sq. ft.
202. $900. 203. $2000. 204. $4744.19.
205. $1125. 206. $219.34.
207. On flour, $729.75; on sugar, $660.25. 208. $2\frac{1}{2}$%.
209. 103355.02 g. 210. $500. 211. $1500.
212. $9\frac{1}{2}$%. 213. $16\frac{128}{145}$ m. 214. $11\frac{1}{4}$ da.
215. 8.421%. 216. $$41.06\frac{2}{3}$. 217. 105.2988 ft.
218. Lost $7\frac{1}{7}$%. 219. $9483.13. 220. $4905.90.
221. Hind-quarter, 136 lb.; fore-quarter, 116 lb.
222. Tea cost 75c. a pound. 223. 41.53 ft.
224. 16. 225. 107.8582 m. 226. $4800.
227. 176.34909 km. per hour. 228. $22.75.
229. $340. 230. 6%. 231. 7.
232. 30.47943%. 233. $1.80 per yard.
234. Sold 548 bbl. flour; purchased 16100 lb. butter.
235. First company loses $2495; second, $7425.
236. 2 yr. 4 mo. 237. 6%. 238. $119\frac{3}{4}$.
239. *A* gets $168; *B*, $171. 240. $22\frac{2}{9}$%.
241. 3. 242. 56.7932%. 243. $27\frac{1}{2}$%.
244. $4023. 245. 3 mo. 246. 5%.
247. 5%. 248. *A* gets $99; *B*, $33; *C*, $22.
249. $\frac{3}{11}$ g. 250. $3\frac{3}{5}$c. 251. 7.0993%.
252. $5\frac{5}{9}$%. 253. $3.8556. 254. $1071.30.
255. 25.92%. 256. 63 marks. 257. $15.
258. $20000. 259. $420; 5%. 260. $2190.
261. $142\frac{2}{9}$ yd. 262. 50%. 263. $5400.
264. 5%. 265. 5 yd. cloth, 9 yd. lining.

266. 1.75635 ac. 267. 13 mills.
268. Gain, $37.30; gain by false weight is $6.05.
269. 13%. 270. $528. 271. $293.33.
272. 6%. 273. *B* furnishes $\frac{5}{14}$ and *C* $\frac{15}{56}$.
274. $7\frac{1}{2}$ mills.
275. $36000 deposited in 1901; $40000 in 1902; $50000 in 1903; and $49000 in 1904. 276. 12 cm.
277. April 3rd. 278. $7800. 279. $4.75.
280. $7900. 281. The square field by 63.222 yd.
282. *A*'s capital is $12000; *B*'s, $15000; *C*'s, $8500.
283. Rates are equal. 284. $13440. 285. $510.62.
286. $1364; 6%. 287. Increase, $15. 288. 51 bu.
289. $56\frac{67}{98}$c. 290. $6\frac{5}{8}\%$. 291. 3 ft. 1.2 in.
292. $831.23. 293. $17200. 294. Jan. 6th, 1891.
295. $11\frac{3}{22}$ ft ; $2728\frac{9}{22}$ sq. ft. 296. .9622995 cu. ft.
297. Increased $\$102\frac{2}{9}$. 298. $1895.71.
299. $112.098. 300. $5234.15. 301. Increased $2.50.
302. *A* gets $1514.13; *B* gets $2708.20. 303. $77000.
304. Net loss, $3550; *A*'s, $1010; *B*'s net insolvency, $2730.
305. 6%. 306. 8%.
307. $4400 in stock at 88. 308. 20 in.
309. 65%. 310. 515.9 yd.
311. *A* gets $1822.50; *B*, $1701; *C*, $1417.50.
312. $2\frac{1}{2}\%$. 313. $1616. 314. $1.60.
315. $2\frac{14}{25}$ hr. 316. $320. 317. $8.
318. Commission = $\$1335\frac{15}{41}$; purchased $4146\frac{14}{41}$ cwt. pork.
319. $33.13 decrease. 320. $1885\frac{5}{7}$ gal. 321. 8 in.
322. *A* has $3555.555; *B* has $5925.925. 323. 10.2%.
324. 7.29%. 325. $4480. 326. $525.25; 4%.
327. $1676.83\frac{1}{3}$. 328. $58\frac{4}{17}\%$. 329. $1426.18.
330. 8225 ft. 331. 9 ft. 332. $1800.
333. $2334.72. 334. 4% stock at par.
335. Average area, $641\frac{2}{3}$ sq. in.; area of circle, 616 sq. in.
336. 7%. 337. 1054.64 g.
338. $3570 and $5950. 339. 6%.
340. 80.486 cu. ft. 341. $3\frac{3}{4}$ ft. 342. $\$44$; $\$44\frac{1}{3}$; $\$46\frac{2}{3}$.

343. 3426.152752 l. 344. 19.3 ... in. 345. 10.46 ... in.
346. 56%. 347. $3.859. 348. $1255.36.
349. 6%. 350. A cow cost $15 and a sheep 1.87\frac{1}{2}$.
351. $1400 in 4%; $4100 in 5%. 352. $445.
353. $5284. 354. 78.558 in. 355. 4687.3035 gr.
356. 16.38631 cu. cm. 357. $2534 in bank stock.
358. $883.74. 359. 2110\frac{10}{63}$; 5$\frac{10}{13}$%; 6$\frac{6}{23}$%.
361. 20.948 ... %. 362. $3760.68. 363. 9 men.
364. $9450. 365. $51834.92. 366. *A* gets $17250.
367. 393.408 sq. in. 368. 43 men. 369. 4777 electors.
370. $\frac{10\sqrt{3}}{3}$; $\frac{10\sqrt{6}}{3}$. 371. 206$\frac{1}{4}$ cu. dcm. 372. $2 per yard.
373. $1460; duty = 243\frac{1}{3}$.
374. $5250; $7000; lost 2$\frac{2}{45}$%. 375. 17.
376. 360 hats. 377. 2264.4 sq. m. 378. 5.88 cu. dcm.
379. $6727. 380. 1956$\frac{22}{63}$ lb. 381. $10000.
382. 2$\frac{1}{2}$%. 383. 1 Kg. = 2.23608 lb.
384. 60c. and 40c. per pound. 385. 15.537 in.
386. 21$\frac{3}{17}$ hr. 387. 16$\frac{4}{5}$ mi. 388. 5%; no.
389. 394200 sheets. 390. He neither loses nor gains.
391. 53.35 l.; 42.68 l.; 35.57 l. 392. $4000.
393. $900. 394. 5% and 7%.
395. 577$\frac{1}{3}$ sq. ft.; 1732$\frac{1}{3}$ sq. ft. 396. 1663.584 sq. m.
397. 498.816 sq. ft. 398. $2606.50. 399. $49.
400. *B*'s stock = 5333\frac{1}{3}$; *C*'s time = 9.6 mo. 401. $510.59.
402. $7995.06. 403. 539$\frac{3}{8}$ sq. ft. 404. 1800 sq. ft.
405. 787.072 sq. ft. 406. 4% at 70 is better investment.
407. 48000 cu. cm.; weight = 48 Kg. 408. 12 min.
409. $2046.98. 410. 7 ft. and 8 ft. 411. 36$\frac{1}{4}$ mi. per hour.
412. 65 men. 413. 833 and 2057. 414. 4$\frac{2}{3}$%.
415. Lost 4%; gained 6$\frac{2}{3}$%. 416. .502 ... ft.
417. Increased $242.42. 418. 19$\frac{1}{2}$ ft.
419. $17530.56. 420. 400 ac. 421. 6$\frac{1}{2}$%; 3$\frac{1}{13}$ yr.
422. Rate of interest is 9.851%; rate of discount is 9.605%.
423. Gain 9.89%. 424. $2300.
425. Life insurance better by $335.26.

426. Velocity of bullet = 900 ft. per second; sound = 1100 ft.
427. $5\frac{26}{35}$%.
428. $\frac{29}{215}$.
429. Loss = $28\frac{8}{9}$%.
430. $600.
431. 72c.
432. $6000.
433. *A* invests $2140; *B*'s time is 12 mo.
434. $38; $1368.
435. 35 min.
436. $15.75.
437. $10000; $5700.
438. $4400.
439. $400.
440. $7700; $9600; 1.73%.
441. 80 mi.
442. $13787.46.
443. $26\frac{2}{3}$%.
444. $9600.
445. 9 p.m.
446. *A* gets $.45; *B*, $.$39\frac{3}{8}$; *C*, $.35; *D*, $.$31\frac{1}{4}$.
447. $19687\frac{1}{2}$ lb.
448. 12 ft.
449. $42\frac{6}{7}$%.
450. $883\frac{13}{14}$ lb.
451. 400 ac.
452. Area = 120 sq. ft.; shortest distance = $9\frac{3}{13}$ ft.
453. $\frac{7\sqrt{3}}{9}$ hr.
454. $300.08.
455. 3% for selling and 4% for buying.
456. $100000.
457. 4 mi. per hour.

PART II.—SOLUTIONS

EXERCISE I. PAGE 11

7. The method of "Equal Additions" depends upon the principle that adding the same number to both minuend and subtrahend does not alter the remainder.

$$\begin{array}{r} 70846 \\ 37298 \\ \hline 33548 \end{array}$$

Hence, as 8 units cannot be taken from 6 units, we add 10 units to 6 units, making 16 units. Then 8 units from 16 units leave 8 units. Now as we added 10 units to the minuend, we must add an equal number to the subtrahend. But 10 units equal 1 ten. Adding 1 ten to 9 tens, we have 10 tens; and as we cannot take 10 tens from 4 tens, we add 10 tens to 4 tens, making 14 tens; then 10 tens from 14 tens leave 4 tens. Now as we added 10 tens to the minuend, we must add 10 tens or 1 hundred to the subtrahend. Then, 2 hundred + 1 hundred = 3 hundred, and 8 hundred − 3 hundred = 5 hundred, etc.

8.

$$\begin{array}{rcccccc} 102110 & = & 9 & (11) & (10) & (10) & (10) \\ 75432 & = & 7 & 5 & 4 & 3 & 2 \\ \cline{3-7} & & 2 & 6 & 6 & 7 & 8 \end{array}$$

9.

$$\begin{array}{r} 50000 \\ \hline 13504 \end{array}$$

This is most readily done by the "Computers' Method of Subtraction." Thus, $8 \times 2 + 4 = 20$; set down 4 and carry 2. $8 \times 6 + 2 = 50$; set down 0 and carry 5; $8 \times 5 + 5 = 45$ and 5 make 50; set down 5 and carry 5; $8 \times 4 + 5 = 37$ and 3 make 40; set down 3 and carry 4; 4 from 5 leaves 1. The remainder is 13504.

10.

$$\begin{array}{lr} & 9643287 \\ & (378)(42)(7) \\ \cline{2-2} \text{7 times the multiplicand} = & 67503009 \\ \left.\begin{array}{l}\text{42 times the multiplicand} = \text{6 times 7 times} \\ \text{the multiplicand} = \text{6 times 67503009}\end{array}\right\} = & 405018054 \\ \left.\begin{array}{l}\text{378 times the multiplicand} = \text{9 times 42} \\ \text{times the multiplicand} = \text{9 times 405018054}\end{array}\right\} = & 3645162486 \\ \cline{2-2} & = 3649280169549 \end{array}$$

12. $616 = 7 \times 8 \times 11.$

$$\begin{array}{r|l l l l l}
7 & \underline{78964} & & & & \\
8 & \underline{11280} & \text{and } 4 \text{ units} & = & 4 & \text{units} \\
11 & \underline{1410} & \text{" } 0 \text{ sevens} & = & 0 & \text{"} \\
 & \ \ 128 & \text{" } 2 \text{ fifty-sixes} & = & \underline{112} & \text{"} \\
 & & & & 116 & \text{"}
\end{array}$$

Quotient = 128; remainder = 116.

14. $762 - 756 = 6.$

Hence $762 \times 908 = 686448 + 6 \times 908 = 691896.$

17. This may readily be seen when the numbers are arranged thus,

47
85 − 47
756 − 85
3691 − 756
4587 − 3691

Adding these together, the sum is evidently 4587.

18. On dividing 57385 by 387 the final remainder is 109. Hence the dividend must be increased by 387 − 109, or by 278 to contain the divisor exactly.

21. Price per foot frontage $= \$\frac{4500}{60} = \$75.$

Price of strip 1 ft. wide and 121 ft. deep is $75.

Price of 1 ac. or 43560 sq. ft $= \$\dfrac{43560 \times 75}{121} = \$27000.$

22. Wages for 42 da. $= 42 \times \$1.80 = \$75.60.$

Loss for each idle day $= \$1.80 + 45\text{c.} = \$2.25.$

Loss through being idle $= \$75.60 - \$64.35 = \$11.25.$

Number of days of idleness $= \$11.25 \div \$2.25 = 5.$

Number of days of working $= 42 - 5 = 37.$

23. Value of 10000 bu. of wheat at 85c. = $8500.

Actual value of wheat and oats = $8028.80.

Excess of value when all is wheat = $471.20.

Each bushel of oats reduces the cost by 40c.;

∴ number of bushels of oats bought $= \frac{47120}{40} = 1178;$

∴ " " " " wheat " $= 10000 - 1178 = 8822.$

31. $(8376+5684)\times(8376-5684)\div7859=4816$ and remainder 576.

$$7859-576=7283.$$

33. If the sum of two numbers is added to the difference of these numbers, the result is equal to twice the greater number;

$$\therefore \text{ the greater number}=\frac{2\times4331+2\times3353}{2}=7684.$$

37.
$$19+17+15=51;$$

$$\therefore \text{ sum received by first out of } \$51=\$19.$$

$$\text{" \quad " \quad } \$35700=\frac{35700\times\$19}{51}$$

$$=700\times\$19$$

$$=\$13300.$$

38. In working such problems as this, begin with the last result and work towards the beginning of the example, always reversing the operation described in the problem.

Arith. Comp. of $7846=10000-7846=2154.$
" " " $3479=10000-3479=6521.$
Difference $=4367.$
Number $=[\{(883+4367)\div10\}+1225]\times4=7000.$

42. Cost of cattle per head $=\$18+\$2=\$20.$

$$\text{Number bought} = \frac{6400}{20}=320.$$

$$\text{Number sold} = \frac{3600}{18}=200.$$

Hence, the difference, 120 cattle, must sell for
$(\$6400-\$3600+\$800)=\$3600.$

43. (*a*) To multiply by 5, multiply by 10 and divide the product by 2. $5=10\div2.$

(*b*) To multiply by 25, multiply by 100 and divide the product by 4. $25=100\div4.$

(*c*) To multiply by 125, multiply by 1000 and divide the product by 8. $125=1000\div8.$

47. Indebtedness $= \frac{7770 \times \$100}{37} = \$21000.$

Sum due creditor $= \frac{1998 \times \$100}{37} = \$5400.$

49. Use (1728)(144)(12) as the multiplier and multiply by 12; then multiply this product by 12 and the new product by 12; add the three partial products together.

53.
$$\begin{array}{rl} & 64 \times 39 + 36 \times 39 = 100 \times 39; \\ \text{and} & 100 \times 39 \div 100 = 39. \end{array}$$

54.
$$\begin{array}{c} 1000 - 482 = 518; \\ 482 \times 689 + 518 \times 689 = 1000 \times 689. \end{array}$$

66.
$$\begin{array}{rl} 57875 & \\ \underline{(729)(81)(9)} & \\ 520875 & = 9 \times 57875 \\ 4687875 & = 9 \times 520875 \\ \underline{42190875} & = 9 \times 4687875 \\ 42238274625 & \end{array}$$

$$\begin{array}{r|l} 9 & \underline{123456} \\ 7 & \underline{13717} \text{ and 3 units over} \\ & 1959 \text{ and 4 groups of 9 units each over;} \end{array}$$

$$\therefore \text{ quotient} = 1959\tfrac{39}{63}.$$

67. M has 12 mi. start. N gains 4 mi. per hour, and hence would overtake M in 3 hr. When N arrives M has 4 × 6 mi. to go. It requires $N \frac{4 \times 6}{4}$ hr., or 6 hr. to gain this distance on M. Hence M travels $(5 + 6 + 4)$ hr. and goes 15×6 mi., or 90 mi.

68. Fast train goes 20 mi. farther than slow train in 1 hr. Fast train goes 100 mi. farther than slow train in 5 hr.; ∴ both trains run 5 hr. Distance $= 5(50 + 30)$ mi. or 400 mi.

69.
$$\begin{array}{lll} & \text{Toll on } 240 \text{ hhd.} & = 2 \text{ hhd.} + \$90. \\ & \text{"} \quad 150 \text{ hhd.} & = 2 \text{ hhd.} - \$30; \\ \therefore & \text{"} \quad 90 \text{ hhd.} & = \$120; \\ \therefore & \text{"} \quad 150 \text{ hhd.} & = \$\frac{150 \times 120}{90} = \$200; \\ \therefore & \text{value of } \; 2 \text{ hhd.} & = \$200 + \$30 = \$230; \\ \therefore & \text{"} \quad 1 \text{ hhd.} & = \$\frac{230}{2} = \$115. \end{array}$$

71. For every 3 da. of the time he earned \$(2 × 1.50) and paid 60c., or his net earning was \$2.40.

Time he took to earn \$2.40 = 3 da.;

$$\therefore \quad " \quad " \quad " \quad \$72 = \frac{72 \times 3}{2.40} \text{ da.} = 90 \text{ da.}$$

72.

$$\begin{array}{rl} 121711 & \\ \underline{(32)(8)(56)} & \\ 973688 & = 800 \text{ times the multiplicand.} \\ 6815816 & = \;\; 7 \text{ times } 8 \text{ times } 121711. \\ \underline{3894752} & = \;\; 40 \text{ times } 800 \text{ times } 121711. \\ 3998936616 & \end{array}$$

73. *A* runs 100 yd. while *B* runs 96 yd.;
A runs 100 yd. while *C* runs 95 yd.;
∴ *B* runs 96 yd. while *C* runs 95 yd.;
hence *B*, giving *C* 1 yd. start, will overtake him at the end of 96 yd., and will therefore beat him in a hundred yards' race.

74. $(1\text{st})^2 \times (2\text{nd})^2 \times (3\text{rd})^2$

$$\begin{aligned} &= 176382 \times 279152 \times 215496; \\ \therefore \; 1\text{st} \times 2\text{nd} \times 3\text{rd} &= \sqrt{(176382 \times 279152 \times 215496)} \\ &= 2 \times 8 \times 9 \times 41 \times 73 \times 239; \\ \therefore \; 1\text{st number} &= \frac{2 \times 8 \times 9 \times 41 \times 73 \times 239}{215496} = 478. \end{aligned}$$

78. Minuend + subtrahend = 1106450.

$$\begin{aligned} \text{Minuend} + \text{subtrahend} &= 1106450. \\ " \quad - \quad " \quad &= \;\; 736858; \\ \therefore \; 2 \text{ times minuend} &= 1843308; \\ \therefore \quad " \quad &= \;\; 921654; \\ \text{also, } 2 \text{ times subtrahend} &= 1106450 - 736858 = 369592; \\ \therefore \quad " \quad &= 184796. \end{aligned}$$

79. Writing the example as in division, as follows:—

$$\begin{array}{r} 529565 \\ \hline 2466 \\ \hline 2225 \\ \hline 542 \end{array}$$

it is readily seen that 5049 must be the first product of the divisor and partial quotient; that 2244 must be the second such product, and that 1683 must be the third.

Hence, there are 3 figures in the quotient, and the divisor must be a common factor of 5049, 2244, and 1683. This is readily found to be 561, and the figures of the quotient are 9, 4, and 3.

Hence, the divisor is 561, and quotient 943.

80. Length of fence $= 2(25+17)$ rd. $= 84$ rd.

Weight of fencing $= \frac{84 \times 255}{20}$ lb. $= 1071$ lb.

Cost of fencing $= 84 \times 63$c. $= \$52.92$.

81. Length of fence $= 2$ mi. $= 10560$ ft.

Weight of fencing $= \frac{3 \times 10560}{16}$ lb. $= 1980$ lb.

Cost of fencing $= \$\frac{1980 \times 3}{100} = \$59.40$.

EXERCISE II. PAGE 22

4. $360 = 2 \times 2 \times 3 \times 3 \times 5$

1, 2, 4, 8
1, 3, 9

1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, 72
1, 5

1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, 72, 5
10, 20, 40, 15, 30, 60, 120, 45, 90, 180, 360.

These are all the possible divisors of 360.

6. (*a*) If no exact quotient is found until the quotient becomes less than the divisor, it is obvious that the number must be a prime one.

Thus if 3407 is divided by successive prime numbers, 2, 3, 5, 7, 11, 13, etc., until 59 is reached, it is clear that it is useless to go beyond 59, for the quotients now obtained have already been tried as divisors.

7. $36288 = 4 \times 4 \times 4 \times 9 \times 9 \times 7$;

$\therefore$ if 36288 is multiplied by 7 it becomes a perfect square.

16. Any four consecutive numbers must be of the form n, $n+1$, $n+2$, $n+3$, where n is either odd or even.

If n is odd, $n+1$ is even, and hence, exactly divisible by 2.

If n is even, then it is exactly divisible by 2.

Again, n, or $n+1$, or $n+2$, must be exactly divisible by 3.

Again, n, or $n+1$, or $n+2$, or $n+3$, must be exactly divisible by 4.

Hence, the number formed by these factors must be exactly divisible by $2 \times 3 \times 4$.

19. The prime factors of 10290 are 2, 3, 5, and 7^3.

Hence, the factors required are 7×2, 7×3, and 7×5, or 14, 21, and 35.

20. Resolve the number into its prime factors. Form as many series as there are different prime factors, making 1 the first term of each series; the first power of the prime factor the second term; the second power of that factor the third term, etc. Multiply these series together.

Prime factors of 8100 = 2, 2, 3, 3, 3, 3, 5 and 5.

1st series = 1, 2, 4.
2nd " = 1, 3, 9, 27, 81.
3rd " = 1, 5, 25.

1, 3, 9, 27, 81
1, 2, 4

1, 3, 9, 27, 81, 2, 6, 18, 54, 162, 4, 12, 36, 108, 324
1, 5, 25

1, 3, 9, 27, 81, 2, 6, 18, 54, 162, 4, 12, 36, 108, 324
5, 15, 45, 135, 405, 10, 30, 90, 270, 810, 20, 60, 180
540, 1620, 25, 75, 225, 675, 2025, 50, 150
450, 1350, 4050, 100, 300, 900, 2700, 8100.

33. The difference between 3961 and 3029 is 932.

$$932 = 4 \times 233.$$

It is evident that 4 is not a common factor; hence, as 233 is a prime number, if these numbers have a common factor it must be 233. By trial, 3961 is found to contain 233, 17 times; hence

$$3029 \div 233 = (17 - 4).$$

36. This is asking to find the H. C. F. of (8095 − 20), (11573 − 23), and (16697 − 22).

39. The example is evidently the following :—

```
1856)4466(2
     3712
      754)1856(2
          1508
           348)754(2
               696
                58)348(6
                   348
```

41. We must here find the 3 smallest, and also the 3 largest numbers that will exactly divide 600.

The prime factors of 600 = 2, 2, 2, 3, 5 and 5;

∴ the 3 smallest bags must hold 1 bu., 2 bu., or 3 bu., and the 3 largest bins, 300 bu., 200 bu., or 150 bu.

46. The L. C. M. of 24, 36, and 32 is 288; hence, 288 − 1 is the number required.

48.

$$\begin{aligned}1260 &= 2 \times 2 \times 3 \times 3 \times 5 \times 7.\\ 36 &= 2 \times 2 \times 3 \times 3.\\ 12 &= 2 \times 2 \times 3.\end{aligned}$$

Hence, the factors of the other number must be 2, 2, 3, 5, and 7. Number is 420.

52. The prime factors of 2772 = 2, 2, 3, 3, 7, and 11. The required numbers must be divisible by 12, and have their L. C. M. 2 × 2 × 3 × 3 × 7 × 11.

∴ one number = 12 × 3 = 36.
∴ a second = 12 × 7 = 84.
∴ a third = 12 × 11 = 132.

55. Time required by first horse to go once round

$$= \frac{5280}{440} \text{ min.} = 12 \text{ min.}$$

Time required by second horse to go once round

$$=\frac{5280}{352}\text{ min.}=15\text{ min.}$$

Time required for third horse to go once round

$$=\frac{5280}{264}\text{ min.}=20\text{ min.}$$

Time required = L. C. M. of 12 min., 15 min., and 20 min.
= 60 min.

59. Distance gone = (360 × 11 × 13) ft.

$$=\frac{360\times 11\times 13}{5280}\text{ mi.}=9\tfrac{3}{4}\text{ mi.}$$

61. We are required to find the H. C. F. of (10974 − 54) and (15336 − 36).

The H. C. F. of 10920 and 15300 = 60.

62. Since 2 is in the units' place, the remainder = 3. So that the subtraction may be completed 1 must be borrowed from the 7 in the millions' place, thus the remainder in the millions' place = 6.

63. Length of avenue = 3 × 5280 ft. = 15840 ft.
L. C. M. of 6, 8, 9, 10, and 12 = 360.

Number of times there are 5 trees in a row $=\frac{3\times 5280}{360}=44.$

65. L. C. M. of 5, 6, and 7 is 210. Hence, least number of marbles there can be is 214. But this number does not fulfil the second condition of the problem.

210 ÷ 11 leaves a remainder of 1. As we have 4 to add, the remainder must be seven in order that the number + 4 will be divisible by 11. Hence, we must have 7 × 210 + 4.

Hence, the number of marbles is 1474.

67. *B* goes 800 while *A* goes 880;
then *B* " 960 " *A* " $\frac{9}{10}$ of 960, or 864;
∴ *B* wins by 16 yd.

68. The L. C. M. of 32 and 40 is 160. Hence, when the small wheel has made 5 revolutions, the larger one has made 4, and the same cogs are together.

No. of revolutions of small wheel in a day $= 10 \times 60 \times 60 \times 64$;

$\therefore$ No. of times two cogs are together $= \dfrac{10 \times 60 \times 60 \times 64}{5}$

$= 460800.$

73. The common factors of 9000 and 6840 are 2^3, 3^2, 5. To have 27 as remainder the divisor must be more than 27. The least common divisor greater than 27 is $2 \times 3 \times 5$, or 30. The G. C. M. is $2^3 \times 3 \times 3 \times 5$, or 360.

[illegible] divisions are :—

$2 \times 5 \times 4$, or 40
$5 \times 3 \times 4$, or 60
$3 \times 3 \times 4$, or 36
$2 \times 5 \times 3 \times 3$, or 90
$2 \times 5 \times 3 \times 4$, or 120
$2 \times 3 \times 3 \times 4$, or 72
$5 \times 3 \times 3$, or 45
$3 \times 3 \times 5 \times 4$, or 180;

or, find all the divisors of 360 (see example 20, page 55 of Key) and reject all less than 30.

74. *A* walks a mile in 16 min.; *B*, in 24 min.; *C*, in 28 min.; and *D*, in 32 min.

L. C. M. of 16 min., 24 min., 28 min., and 32 min., is 672 min. Now, as each spends half of 672 in going and half in returning;

$\therefore$ distance *A* goes $= \frac{336}{16}$ mi. $= 21$ mi.
" *B* " $= \frac{336}{24}$ mi. $= 14$ mi.
" *C* " $= \frac{336}{28}$ mi. $= 12$ mi.
" *D* " $= \frac{336}{32}$ mi. $= 10\frac{1}{2}$ mi.

75. L. C. M. of 5 and 4 is 20.

Cost price of 20 oranges $= 4 \times 3$c. $= 12$c.
Selling " 20 " $= 5 \times 3$c. $= 15$c;
$\therefore$ number on which 3c. are gained $= 20$;
$\therefore$ " " \$1.35 " " $= \dfrac{135 \times 20}{3}$

$= 900.$

EXERCISE III. PAGE 33

14. (*b*) (3) $$\frac{\frac{1}{21}\times 5\frac{17}{23}\times 6\frac{3}{11}+6\frac{19}{51}\times 1\frac{23}{49}\div 2\frac{5}{17}+1\frac{10}{49}}{9\frac{16}{57}\times 1\frac{22}{23}\div 5\frac{17}{38}+3\frac{11}{78}\times 6\frac{17}{21}\div 7\frac{21}{32}}\times 12\frac{4}{9}$$

$$=\frac{\frac{1}{21}\times\frac{132}{23}\times\frac{69}{11}+\frac{325}{51}\times\frac{72}{49}\times\frac{17}{39}+\frac{59}{49}}{\frac{529}{57}\times\frac{45}{23}\times\frac{38}{207}+\frac{245}{78}\times\frac{143}{21}\times\frac{32}{245}}\times\frac{112}{9}$$

$$=\frac{\frac{12}{7}+\frac{200}{49}+\frac{59}{49}}{\frac{10}{3}+\frac{176}{63}}\times\frac{112}{9}=\frac{\frac{343}{49}}{\frac{386}{63}}\times\frac{112}{9}$$

$$=\frac{343}{49}\times\frac{63}{386}\times\frac{112}{9}=\frac{2744}{193}=14\frac{42}{193}.$$

(4) $$\frac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times 3\frac{33}{34}\div 3\frac{37}{96}}{8\frac{17}{19}\times 5\frac{14}{39}\div 4\frac{15}{32}-7\frac{19}{20}\times 5\frac{11}{65}\div 14\frac{21}{25}}\times 4\frac{8}{23}$$

$$=\frac{\frac{1}{23}\times\frac{115}{17}\times\frac{323}{13}-\frac{85}{18}\times\frac{135}{34}\times\frac{96}{325}}{\frac{169}{19}\times\frac{209}{39}\times\frac{32}{143}-\frac{159}{20}\times\frac{336}{65}\times\frac{25}{371}}\times\frac{100}{23}$$

$$=\frac{\frac{95}{13}-\frac{72}{13}}{\frac{32}{3}-\frac{36}{13}}\times\frac{100}{23}=\frac{1}{13}\times 23\times 39\times\frac{1}{308}\times\frac{100}{23}=\frac{75}{77}.$$

(5) $$\frac{19}{7\times\frac{2}{3-1\frac{2}{3}}}\times\frac{7735}{67184}\div\left(1\frac{3}{16}-\frac{47}{48}\right)$$

$$=\frac{19}{7\times\frac{3}{2}}\times\frac{7735}{67184}\times\frac{48}{10}=\frac{19}{7}\times\frac{2}{3}\times\frac{7735}{67184}\times\frac{48}{10}=1.$$

(6) $$\frac{1}{2+\frac{3}{4+\frac{5}{6}}}\times\frac{4862}{4147}\div\left(1\frac{1}{2}-\frac{23}{38}\right)=\frac{29}{76}\times\frac{4862}{4147}\times\frac{38}{34}=\frac{1}{2}.$$

EXERCISE IV. PAGE 36

3. $\frac{4}{7}$ of $\frac{5}{11}$ of share = \$3600;

∴ whole = $\frac{7}{4}$ of $\frac{11}{5}$ of \$3600 = \$13860.

4. Since the sum of two numbers added to their difference = twice the greater, we have

$$4\frac{1}{5}+2\frac{4}{7}=\frac{237}{35}=\text{twice the greater};$$

$$\therefore\ \text{greater}=\frac{237}{70}=3\frac{27}{70}.$$

And $4\frac{1}{5}-3\frac{27}{70}=\frac{57}{70}=$ the less.

5. The first number is to be made the numerator, and the second number the denominator of the same fraction.

$$\begin{aligned} &\text{Since} & 1 \text{ is } &\frac{1}{9\frac{1}{5}} \text{ of } 9\tfrac{1}{5}; \\ &\therefore\ 3\tfrac{1}{2} \text{ is } & 3\tfrac{1}{2}\times &\frac{1}{9\frac{1}{5}} \text{ of } 9\tfrac{1}{5}, \\ &\text{or} & &\frac{3\frac{1}{2}}{9\frac{1}{5}} \text{ of } 9\tfrac{1}{5}, \\ &\text{or} & &\frac{35}{92} \text{ of } 9\tfrac{1}{5}. \end{aligned}$$

6. The relative magnitudes will be obvious when the fractions are reduced to the same denominator.

7. The sum of the fractions is $\frac{3}{11}$.

$$\frac{3}{11}=\frac{3000}{11000}=\frac{272+}{1000}, \text{ next less than } \frac{273}{1000};$$

and $1-\frac{272}{1000}=\frac{728}{1000}$, the fraction required.

9. $\frac{3}{8}$ of value of ship $=\frac{1}{4}$ value of cargo;
$\therefore$ value of ship $=\frac{8}{3}$ of $\frac{1}{4}$ or $\frac{2}{3}$ value of cargo;
$\therefore$ $\frac{5}{3}$ value of cargo $=$ \$60000.

10. The denominator, *i.e.*, the "*name-giver*," because it gives the name to the parts.

The numerator, *i.e.*, the "*numberer*," or "*counter*," because it indicates how many of the parts named by the denominator are to be taken.

12.
$$\tfrac{1}{2}=\tfrac{2}{4}=\tfrac{3}{6}=\tfrac{4}{8}=\tfrac{5}{10}=\tfrac{6}{12};$$

$$\text{then } \tfrac{1}{2}=\frac{1+2+3+4+5+6}{2+4+6+7+10+12}=\tfrac{21}{42}=\tfrac{1}{2}.$$

14. *C*'s age is evidently 84 yr.
B's " $=\frac{4}{7}$ of *C*'s $=\frac{4}{7}$ of $84=48$ yr.
A's " $=\frac{5}{12}$ of *B*'s $=\frac{5}{12}$ of $48=20$ yr.

15. In the operation of addition of integers, the addends must have the same name, in order that their sum may be expressed

by one number; so also in fractions, the addends must have the same fractional unit in order that their sum may be expressed as one fraction.

17. If operations, the reverse of those indicated in the question, be performed on $\frac{2}{3}$, the required number will be found; hence,

$$\left\{\left(\tfrac{2}{3}+\tfrac{1}{2} \text{ of } \tfrac{3}{7} \text{ of } 14\tfrac{4}{5}\right)\div\frac{2\frac{1}{2}}{3}-2\tfrac{3}{4}\right\}\times 8\tfrac{4}{17}$$
$$=\left(\tfrac{79}{21}\times\tfrac{6}{5}-2\tfrac{3}{4}\right)\times 8\tfrac{4}{17}$$
$$=\tfrac{247}{140}\times 8\tfrac{4}{17}$$
$$=14\tfrac{9}{17}.$$

18. Carriage $=\frac{7}{8}$ of horse;

∴ horse $+\frac{7}{8}$ of horse $=\$225$.

$\frac{15}{8}$ value of horse $=\$225$;

∴ value of horse $=\$\dfrac{8\times 225}{15}=\$120$.

Carriage $=\$225-\$120=\$105$.

Harness $=\dfrac{2\frac{1}{2}}{12}$ of $\$120=\$25$.

19. Let 1 share represent *B*'s share, then

since *B*'s = 1 share

A's = 3 shares − $88,

and *C*'s = 2 shares − $44 + $176,

and *A*'s + *B*'s + *C*'s = 6 shares + $44 = $8888;

∴ 6 shares = $8844.

1 share = $1474 = *B*'s share.

A's share = 3 × $1474 − $88 = $4334.

C's share = 2 × $1474 + $132 = $3080.

23. After spending $80 less than $\frac{2}{3}$ of his money, John has $\frac{1}{3}$ of his money + $80 left; then if 1 represents his money, we have

$$\left(\tfrac{1}{3}+\$80\right)-\left\{\tfrac{2}{7}\left(\tfrac{1}{3}+\$80\right)+\$40\right\}=\$40.$$
$$\tfrac{5}{7}\left(\tfrac{1}{3}+\$80\right)-\$40=\$40,$$
$$\text{or, } \tfrac{1}{7}\left(\tfrac{1}{3}+\$80\right)=\$20.$$
$$\tfrac{1}{3}+\$80=\$140.$$
$$\tfrac{1}{3}=\$60;$$

∴ whole of his money $=\frac{3}{3}=\$180$.

24. $\frac{1}{4}$ of $\frac{2\frac{1}{3}}{7}=\frac{1}{12}$; $\frac{2}{3}$ of remaining $\frac{11}{12}$ or $\frac{11}{18}$ is in the water. Hence, in mud and water there is $\frac{1}{12}+\frac{11}{18}=\frac{25}{36}$; and in air, $1-\frac{25}{36}=\frac{11}{36}$, which $=5\frac{1}{2}$ ft.;

$$\therefore \text{ whole post, or } \frac{36}{36}=\frac{36\times 11}{11\times 2}=18 \text{ ft.}$$

25. Art. 11, page 34.

30. To obtain the product of the multiplier and multiplicand, we perform the same operation on the multiplicand as we did on unity to obtain the multiplier.

Thus, to multiply $\frac{2}{3}$ by $\frac{3}{4}$, what was done with 1 to make $\frac{3}{4}$, the same must be done with $\frac{2}{3}$. But to make $\frac{3}{4}$, 1 is divided into 4 equal parts, and three of them are taken. Hence, to make $\frac{2}{3}$ multiplied by $\frac{3}{4}$, $\frac{2}{3}$ must be divided into 4 equal parts, and 3 of them must be taken.

31. Wife and son had $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$; daughter had, therefore, $1-\frac{5}{6}=\frac{1}{6}$. Mother leaves $\frac{3}{5}$ of $\frac{1}{3}=\frac{1}{5}$ to son, and rest, $\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$, to daughter; daughter then had $\frac{1}{6}+\frac{2}{15}=\frac{3}{10}$. Son's and daughter's shares make the whole, and of this daughter gets $\frac{1}{3}$. Hence, daughter's gain $=\frac{1}{3}-\frac{3}{10}=\frac{1}{30}$.

32. Number that can read $=\frac{1}{2}$
" " " " and write $=\frac{21}{50}$ of $\frac{1}{2}=\frac{21}{100}$
" " " " write and cipher $=\frac{4}{25}\{1-(\frac{1}{2}+\frac{21}{100})\}$
$=\frac{29}{625}$.
The rest $=1-(\frac{1}{2}+\frac{21}{100}+\frac{29}{625})=\frac{609}{2500}=243600$;
$\therefore$ the whole population $=\frac{2500}{609}\times 243600$
$=1000000$.

34. *A* now owns $\frac{5}{9}-\frac{3}{4}$ of $\frac{5}{9}=\frac{5}{9}(1-\frac{3}{4})=\frac{5}{36}$.
B " " $\frac{5}{12}-\frac{4}{7}$ of $\frac{5}{12}=\frac{5}{12}(1-\frac{4}{7})=\frac{5}{28}$.
C " " $\frac{5}{21}-\frac{9}{11}$ of $\frac{5}{21}=\frac{5}{21}(1-\frac{9}{11})=\frac{10}{231}$.
D " " $\frac{9}{11}$ of $\frac{5}{21}$ $=\frac{15}{77}$.

35. Operations are more easily performed.

37. Let 1 denote number of sovereigns, then

$$1-\left(\tfrac{1}{6}+\tfrac{2}{9}+\tfrac{1}{12}+\tfrac{2}{9}+\tfrac{3}{18}\right)=5,$$

$$\text{or } \tfrac{5}{36}=5;$$

$$\therefore\ 1=36.$$

Hence, whole number of sovereigns = 36.

38. Let *A*, *B*, *C* represent the horses.

A would go round the race course in 2 min.
B " " " $2\frac{1}{2}$ min.
C " " " 3 min.

The L. C. M. of 2, $2\frac{1}{2}$, 3, is 30; hence in 30 min. they will be together.

39. $13\frac{1}{3}\div\{(2\frac{6}{7}-2\frac{8}{11})\times 1\frac{4}{7}\}=\frac{40}{3}\div(\frac{10}{77}\times\frac{11}{7})=\frac{40}{3}\times\frac{77}{10}\times\frac{7}{11}=\frac{196}{3}.$

$13\frac{1}{3}\div\ (2\frac{6}{7}-2\frac{8}{11})\times 1\frac{4}{7}\ =\frac{40}{3}\div\ \frac{10}{77}\times\frac{11}{7}\ =\frac{40}{3}\times\frac{77}{10}\times\frac{11}{7}=\frac{484}{3}.$

$13\frac{1}{3}\div\ \ 2\frac{6}{7}-2\frac{8}{11}\ \times 1\frac{4}{7}\ =\frac{40}{3}\times\ \frac{7}{20}-\frac{30}{11}\ \times\frac{11}{7}\qquad=\frac{8}{21}.$

$(\frac{484}{3}-\frac{196}{3})\div\frac{8}{21}=252.$

40. $\frac{2}{3}$ length of longer $=\frac{3}{4}$ length of shorter;
∴ " " $=\frac{9}{8}$ " "
∴ length of both $=\frac{17}{8}$ " "
$=136$ ft.
∴ length of shorter $=\frac{8}{17}$ of 136 ft.;
$=64$ ft.

42. $\frac{2}{25}$ is represented by 1;
∴ 1 " " " $\dfrac{1}{\frac{2}{25}}$
∴ $\frac{4}{35}$ " " " $\frac{4}{35}\times\frac{25}{2}$, or $1\frac{3}{7}$.

43. The English $=\frac{12}{40}$ of regiment
$=\frac{9}{40}$ of (regiment + 450 men);
∴ $\frac{12}{40}$ of regiment $=\frac{9}{40}$ of (regiment + 450 men);
∴ $\frac{3}{40}$ of regiment $=\frac{9}{40}$ of 450 men;
∴ regiment $=\frac{40}{3}$ of $\frac{9}{40}$ of 450 men
$=1350$ men.

EXERCISE V. PAGE 40

8. (*a*) In reducing a vulgar fraction to a decimal, the successive figures in the decimal are found by multiplying the successive remainders by 10. The factors of 10 are two and five. Hence, those vulgar fractions in their lowest terms produce terminating decimals in which the prime factors of the denominator are all twos, or all fives, or all twos and fives.

Such fractions produce pure recurring decimals when there are no twos or fives among the prime factors of the denominators.

Such fractions produce mixed recurring decimals when there are twos or fives and other numbers among the prime factors of the denominators.

(*b*) $32 = 2^5$. Hence, $\frac{5}{32}$ produces terminating decimals of five places.

$12 = 2^2 \times 3$. Hence, $\frac{7}{12}$ produces a mixed repetend consisting of two figures in the non-repeating part and the repeating part.

$24 = 2^3 \times 3$. Hence, $\frac{19}{24}$ produces a mixed repetend consisting of three figures in the non-repeating part and the repeating part.

$56 = 2^3 \times 7$. Hence, $\frac{47}{56}$ produces a mixed repetend consisting of three figures in the non-repeating part and the repeating part.

$\frac{5}{13}$ produces a pure repetend.

$26 = 2 \times 13$. Hence, $\frac{7}{26}$ produces a mixed repetend consisting of one figure in the non-repeating part and the repeating part.

10. The method of performing arithmetical operations with Recurring Decimals will be best explained by taking the operations separately.

(*a*) ADDITION.

Find the sum of $3.4\dot{9}$, $4.\dot{0}4\dot{7}$, and $.14\dot{6}\dot{3}$.

If absolute accuracy is required, proceed as follows :—

First make the repetends *similar*, i.e., make them all begin to repeat in the same place to the right of the point. (See (1) below.)

Next, make the repetends *coterminous*, i.e., make them all end in the same place to the right of the point.

This is done by finding the L. C. M. of 1.3.2, the number of figures in each repetend (2) below.

$$\begin{array}{rclcl} & & (1) & & (2) \\ 3.49\dot{9} & = & 3.49\dot{9} & = & 3.49\dot{9}99999\dot{9} \\ 4.\dot{0}4\dot{7} & = & 4.04\dot{7}0\dot{4} & = & 4.04\dot{7}0470\dot{4} \\ .14\dot{6}\dot{3} & = & .14\dot{6}\dot{3} & = & .14\dot{6}36363\dot{3} \\ \hline & & & & 7.69\dot{3}4106\dot{8} \end{array}$$

In ascertaining how many are to be carried, add up the first column of repetends, thus, 6 + 7 + 9 = 22, carry 2. Then, 3 + 4 + 9 + 2 = 18, etc.

(*b*) SUBTRACTION.

Here we proceed on the same principle as in Addition.

Thus to subtract $5.24\dot{7}$ from $8.\dot{0}5\dot{9}$,

$$\begin{array}{rclcl} 8.\dot{0}5\dot{9} & = & 8.05\dot{9}0\dot{5} & = & 8.05\dot{9}0\dot{5} \\ 5.24\dot{7} & = & 5.24\dot{7} & = & 5.24\dot{7}7\dot{7} \\ \hline & & & & 2.81\dot{1}2\dot{8} \end{array}$$

11. In *Multiplication* and *Division* the recurring decimals should be converted into vulgar fractions, and when the product or quotient of these fractions has been found, it may be converted into a decimal.

$$\begin{aligned} \text{Thus, } 4.\dot{5} \times 3.\dot{7} &= \frac{45-4}{9} \times \frac{37-3}{9} \\ &= \tfrac{41}{9} \times \tfrac{34}{9} = \tfrac{1394}{81}, \\ \text{and } .0\dot{5} \div .04\dot{2} &= \tfrac{5}{90} \div \tfrac{38}{900} \\ &= \tfrac{5}{90} \times \tfrac{900}{38} = \tfrac{25}{19}. \end{aligned}$$

We may now, if it be required, convert $\frac{1394}{81}$ and $\frac{25}{19}$ into decimals.

EXERCISE VI. PAGE 43

1. Take 123.4567 for example and multiply it by 1000.

$$\begin{aligned}123.4567 \times 1000 &= \tfrac{1234567}{10000} \times \tfrac{1000}{1}\\ &= \tfrac{1234567}{10}\\ &= 123456.7.\end{aligned}$$

Hence to multiply by 1000 is to move the decimal point three places to the right.

2. If the divisor be a decimal, change it into an integer by removing the point a sufficient number of places to the right, and also remove the point in the dividend the same number of places to the right. Divide as in the case of integers.

3. By actual division, $\frac{10}{9.009} = 1.11\frac{1}{9.009}$; and

" " $\frac{10}{1.11} = 9.009\frac{1}{1.11}$;

and since $\frac{1}{9.009}$ is less than $\frac{1}{1.11}$, evidently the first statement is more nearly correct.

5. $\frac{355}{113} = 3.1415929$; hence, limits of error lies between .0000006 and .0000009.

10. The *advantages* are (*a*) that the addition, subtraction, multiplication and division of decimals can be performed by processes the same as in ordinary whole numbers, with only additional rules for placing the decimal points in the results; and (*b*) decimals can be compared with the same ease as whole numbers, whereas vulgar fractions have to be reduced to a common denominator.

The *disadvantages* are in recurring decimals, which are only approximations.

14. $.\dot{6}\dot{3} \times .13\dot{6} \times \text{third fraction} = \frac{4}{7}$;

or $\frac{63}{99} \times \frac{123}{900} \times \text{third fraction} = \frac{4}{7}$;

$\therefore$ third fraction $= \frac{11}{7} \times \frac{300}{41} \times \frac{4}{7} = \frac{13200}{2009}$.

15. He sells $\frac{50}{36}$ yd. for \$.50
" " 140 " " \$$\frac{50}{100} \times \frac{36}{50} \times \frac{140}{1}$.
140 yd. at \$.36 cost \$50.40.

16. If $\frac{18}{25} = 423$
$1 = \frac{25}{18} \times 423$
$\therefore \frac{34}{47} = \frac{34}{47} \times \frac{25}{18} \times 423 = 425.$

18. 1 so-called yard measures $(1 - .0208\dot{3})$ yd.
84 " " " $84(1 - .0208\dot{3})$ yd.

19. See page 64 of this book.

22. Divisor + quot. $= 7\frac{1}{2}$
divisor $= \frac{3}{7}$ quot.
rem. $= \frac{20}{27}$ divisor, and
$\therefore = \frac{20}{27}$ of $\frac{3}{7}$ quot. $= \frac{20}{63}$ quot.;
$\therefore \frac{3}{7}$ quot. + quot. $= 7\frac{1}{2}$, or $\frac{10}{7}$ quot. $= 7\frac{1}{2}$;
$\therefore$ quot. $= 5\frac{1}{4}$.
But dividend = quot. × div. + rem.
$= \frac{21}{4} \times \frac{3}{7}$ of $\frac{21}{4} + \frac{20}{63}$ of $\frac{21}{4}$
$= 13\frac{23}{48}$.

23. B's $= A$'s − \$46.70,
C's $= B$'s − \$34.59 = (A's − \$46.70) − \$34.59
= A's − \$81.29.
Sum of all the shares = A's + B's + C's
= A's + A's − \$46.70 + A's − \$81.29
= 3 A's − \$127.99;
$\therefore$ 3 A's − \$127.99 = \$448.715
3 A's = \$576.705
A's = \$192.23$\frac{1}{2}$.

24. In reducing a vulgar fraction in its lowest terms to a decimal, the denominator becomes the divisor. It is evident that there can be only as many different remainders as there are units in the denominator, less one, before the same remainder must occur the second time. Hence, when a fraction in its lowest terms is reduced to a decimal and produces a recurring decimal, the *extreme* limit of the number of places in the *period* of the recurring decimal is one less than the denominator.

Thus $\frac{1}{7}$ produces a recurring decimal of 6 places.
$\frac{3}{19}$ produces a recurring decimal of 18 places.
$\frac{3}{29}$ produces a recurring decimal of 28 places.

25. Sum left after the first spending

$$=\tfrac{11}{55}\text{ of money}-\$2\tfrac{1}{2}$$
$$=\tfrac{1}{5}\text{ of money}-\$2\tfrac{1}{2}.$$

As he spent $\frac{960}{1441}$ of ($\frac{1}{5}$ of money $-\$2\frac{1}{2}$) $-\$1\frac{15}{99}$,
he had remaining $\frac{481}{1441}$ of ($\frac{1}{5}$ of money $-\$2\frac{1}{2}$) $+\$1\frac{15}{99}$;

$$\text{or } =\frac{481}{1441\times 5}\text{ of money}-\$\tfrac{2405}{2882}+\$1\tfrac{15}{99};$$

$$\therefore\ \frac{481}{1441\times 5}\text{ of money}-\$\tfrac{2405}{2882}+\$1\tfrac{15}{99}=\$2\tfrac{603}{990};$$

$$\therefore\ \frac{481}{1441\times 5}\text{ of money}=\$(2\tfrac{603}{990}+\tfrac{2405}{2882}-1\tfrac{15}{99})$$

$$=\$\frac{594516}{9\times 110\times 262};$$

$$\therefore\text{ the money}=\frac{1441\times 5\times 594516}{481\times 9\times 110\times 262}$$
$$=\$34\tfrac{1}{2}.$$

This example illustrates the utility of merely indicating the multiplication and division until the final result is required.

26.
$$\text{Less}=\ .0244\text{ times greater;}$$
$$\therefore\text{ less}+\text{greater}=1.0244\text{ “ “}$$
$$=7.19641;$$
$$\therefore\text{ greater part }=\frac{7.19641}{1.0244}.$$

EXERCISE VIII. PAGE 49.

3. $\frac{.14}{7}=.02.$

$\frac{140}{.07}=\frac{14000}{7}=2000.$

$\frac{.014}{7000}=.000002.$

Sum $=2000.020002=\frac{2000020002}{1000000}=\frac{1000010001}{500000}.$

8. Part sold at cost $= \frac{1}{8}$ of $\frac{1}{5}$, or $\frac{1}{40}$ of goods.

" $\frac{1}{3}$ of cost $= \frac{7}{8}$ of $\frac{1}{5}$, or $\frac{7}{40}$ "

Sum the goods realized $= (\frac{1}{40} + \frac{1}{3}$ of $\frac{7}{40})$ of cost $= \frac{1}{12}$ of cost;

∴ cost of goods $= 12 \times \$1155$,

and loss $= 11 \times \$1155 = \$12705$.

13. $\frac{17}{40} + \frac{8}{19}$ of $\frac{23}{40} = \frac{507}{760}$;

∴ $\frac{253}{760}$ of the ore $= 506$ t.;

$$\therefore \text{ the ore} = \frac{760 \times 506}{253} \text{ t.} = 1520 \text{ t.}$$

14. The net annual increase is 1 in 60, and hence, the population of each year $= \frac{61}{60}$ of the population of the preceding year;

∴ population at end of 5 yr. $= (\frac{61}{60})^5$ of 10000000

$= 10861578$, nearly.

17. Number of killed and wounded

$= \frac{1}{8}$ of $\frac{2}{7}$ of $\frac{1}{3}$ of army $= \frac{1}{84}$ of army;

∴ $\frac{1}{84}$ of army $= 500$ men;

∴ army $= 84 \times 500$ men

$= 42000$ men.

22.
$$\frac{\frac{5}{6} + \frac{11}{45} - \frac{11}{18}}{\frac{43}{7} \times \frac{7}{2} - \frac{7 \times 7 \times 9}{3 \times 2 \times 7} + \frac{8}{5}} = \frac{\frac{7}{15}}{\frac{63}{5}} = \frac{1}{27};$$

$$\frac{40}{55} \times \frac{11}{7} \times \frac{21}{4} + \frac{1}{3} + \frac{17}{27} = 6 + \frac{1}{3} + \frac{17}{27} = 6\tfrac{26}{27};$$

$$\therefore 6\tfrac{26}{27} + \frac{1}{27} = 7.$$

23. Increase of shorter in 100 yr. $= 3.014$;

∴ " " 125 yr. $= \frac{125 \times 3.014}{100}$ in.

$= 3.7675$;

hence, the longer has to increase $(3.7675 - 1.02)$ in., or 2.7475 in.

Increase in 125 yr. $= 2.7475$ in.;

∴ " " 100 yr. $= \frac{100 \times 2.7475}{125}$

$= 2.198$ in.

24. $.041\dot{6} = \frac{10}{240}$ and $.0375 = \frac{9}{240}$;

$\therefore$ $\frac{10}{240}$ of number of inmates at first $= \frac{9}{240}$ of (number of inmates at first $+ 40$),

and $\frac{1}{240}$ of number of inmates at first $= \frac{9}{240}$ of 40;

$\therefore$ number of inmates at first $= 240 \times \frac{9}{240}$ of $40 = 360$;

$\therefore$ number of masters $= \frac{10}{240}$ of $360 = 15$,

and number of boys $= 360 - 15 = 345$.

26. Water admitted in 1 hr. $= 5 \times 3\frac{3}{4}$ t. $= 18\frac{3}{4}$ t.;

$\therefore$ water gains $(18\frac{3}{4} - 12)$ t., or $6\frac{3}{4}$ t. in 1 hr.

Number of hours to gain 60 t. $= \dfrac{60}{6\frac{3}{4}} = 8\frac{8}{9}$ hr.;

$\therefore$ rate of sailing $= \dfrac{40}{8\frac{8}{9}}$ mi. $= 4\frac{1}{2}$ mi.

29. Robert's debt to Charles $= \frac{2}{3}$ of Robert's debt to Charles $+ 10d.$;

$\therefore$ $\frac{1}{3}$ of Robert's debt to Charles $= 10d.$;

$\therefore$ " " " " $= 3 \times 10d. = 2s.\ 6d.$

30. Time he takes to ride 1 way $= \dfrac{2\frac{1}{2}}{2}$ hr. $= 1\frac{1}{4}$ hr.;

$\therefore$ " " " " walk 1 " $= (3\frac{3}{4} - 1\frac{1}{4})$ hr. $= 2\frac{1}{2}$ hr.;

$\therefore$ " " " " " both ways $= 5$ hr.

31. Number of cows $= \frac{1}{7}$ (number of cows + number of sheep);

$\therefore$ 7 times " " " = number of cows + number of sheep;

$\therefore$ 6 " " " " = " " sheep.

Again, number of cows + 3 cows

$= \frac{2}{11}$ of (7 times number of cows + 3 cows);

$\therefore$ 11 times number of cows + 33 cows.

$= 14$ times number of cows + 6 cows;

$\therefore$ 3 " " " $= 27$

$\therefore$ " " $= 9$;

$\therefore$ " " sheep $= 6 \times 9 = 54$.

32. In the fraction $\frac{5}{13}$, the denominator exceeds the numerator by 8;

$\therefore$ equivalent fraction whose denominator exceeds its numerator by 44 is

$$\frac{5 \times \frac{44}{8}}{13 \times \frac{44}{8}} = \frac{27\frac{1}{2}}{71\frac{1}{2}}.$$

34. The sum of quotient and dividend $= 36783 - 208$

$$= 36575;$$

$$\therefore \frac{\text{dividend}}{208} + \text{dividend} = 36575;$$

$$\therefore \quad " \quad = \tfrac{208}{209} \text{ of } 36575$$

$$= 208 \times 175$$

$$= 36400;$$

$\therefore$ number required $= 36400.$

35. Since $\tfrac{4}{5}$ of longer $= \tfrac{8}{9}$ of shorter;

$\therefore$ the " $= \tfrac{5}{4}$ of $\tfrac{8}{9}$ " "

$= \tfrac{10}{9}$ " "

$\therefore \tfrac{10}{9}$ of shorter + shorter $= 95$ ft.;

$\therefore \tfrac{19}{9}$ of " $= 95$ ft.;

$\therefore$ " $= \tfrac{9}{19}$ of 95 ft. $= 45$ ft.,

and longer $= (95 - 45)$ ft. $= 50$ ft.

37. 5 times boy's age = father's age.

But boy's age + 6 yr. $= \tfrac{1}{3}$ of (5 times boy's age + 6 yr.);

$\therefore$ 3 times " " + 18 yr. = 5 times boy's age + 6 yr.;

$\therefore$ 2 times boy's age = 12 yr.;

$\therefore$ " " = 6 yr.

38.

$$\begin{array}{r|l} 11 & 7.00 \\ \hline 2 & .\dot{6}\dot{3} \\ \hline & .3181818\dot{1}\dot{8} \\ & .00003977\dot{7}\dot{2} \\ \hline & .31822159\dot{0} \end{array}$$

the difference lies between .0001 and .0002.

$$\begin{array}{r} 3.14159)1.000000000(.31831\ldots \\ \underline{942477} \\ 575230 \\ \underline{314159} \\ 2610710 \\ \underline{2513272} \\ 974380 \\ \underline{942477} \\ 319030 \\ \underline{314159} \end{array}$$

EXERCISE IX. PAGE 56

5.

$$\frac{(4.5)^3 - (3.4)^3}{4.5 - 3.4} = (4.5)^2 + (4.5)(3.4) + (3.4)^2$$

$$= 20.25 + 15.3 + 11.56$$

$$= 47.11.$$

6. 2, 3, 5, and 7 are prime factors of 12, 15, and 21.

Hence, to be a cube and contain these numbers, the required number must have 2^3, 3^3, 5^3, and 7^3 for its factors;

$$\therefore \text{ number} = 2^3 \times 3^3 \times 5^3 \times 7^3 = 10^3 \times 3^3 \times 7^3 = 9261000.$$

8.

$$\begin{aligned} 6^2 - 5^2 &= 36 - 25 = 11 = 2 \times 6 - 1 \text{ or } 2 \times 5 + 1. \\ 7^2 - 6^2 &= 49 - 36 = 13 = 2 \times 7 - 1 \text{ or } 2 \times 6 + 1. \\ 8^2 - 7^2 &= 64 - 49 = 15 = 2 \times 8 - 1 \text{ or } 2 \times 7 + 1. \\ 11^2 - 10^2 &= 121 - 100 = 21 = 2 \times 11 - 1 \text{ or } 2 \times 10 + 1. \end{aligned}$$

Hence, to find the square of a number from that of the number next above it, subtract from the given square one less than twice the number of which it is the square, and the remainder is the required square:

Or, to find the square of a number from that of the number next below it, add to the given square one more than twice the number of which it is the square, and the sum is the required square.

9.

$$\begin{aligned} &\sqrt{614656} = 784. \\ &783^2 = 614656 - (2 \times 784 - 1) = 613089. \\ &785^2 = 614656 + (2 \times 784 + 1) = 616225. \end{aligned}$$

EXERCISE XIV. PAGE 61

2.

$$\begin{aligned} &2 \text{ No.}^2 = 1270418 \\ \therefore\ &\text{No.}^2 = 1270418 \div 2 = 635209 \\ \therefore\ &\text{No.} = \sqrt{635209} = 797. \end{aligned}$$

8. L. C. M. of 2, 3, 4, 5, 6 = 60.

We must now find the least multiple of 60 which is a perfect square. This is 900.

9.

$$\begin{aligned} 1^3 &= 1 \\ 1^3 + 2^3 &= 9 = (1+2)^2 \\ 1^3 + 2^3 + 3^3 &= 36 = (1+2+3)^2 \\ 1^3 + 2^3 + 3^3 + 4^3 &= 100 = (1+2+3+4)^2 \\ 1^3 + 2^3 + 3^3 + 4^3 + 5^3 &= 225 = (1+2+3+4+5)^2, \text{ etc.}; \\ \therefore\ 1^3 + 2^3 + 3^3 + \ldots + 9^3 &= (1+2+3+\ldots+9)^2; \\ \therefore\ \sqrt{1^3 + 2^3 + 3^3 + \ldots + 9^3} &= \sqrt{(1+2+3+\ldots+9)^2} \\ &= 1+2+3+\ldots+9 = 45. \end{aligned}$$

10. The product of the sum of two numbers by their difference is equal to the difference of their squares;

$\therefore$ the difference of the squares = 12032013.
$2006^2 =$ 4024036;
$\therefore$ square of larger number = 16056049;
and " " $= \sqrt{16056049} = 4007$.

17. Length of side of lot $= \sqrt{2209}$ yd. = 47 yd.
" " fencing = 4×47 yd. = 188 yd.
Weight of " $= \frac{188}{220}$ of 255 lb. $= 217\frac{10}{11}$ lb.

19. Perimeter of square field $= (4 \times \sqrt{193600})$ yd. = 1760 yd.
" " rectangle $= 2(550 + \frac{193600}{550})$ yd. = 1804 yd.
Additional cost $= (1804 - 1760) \times 12$c. = $5.28.

EXERCISE XV. PAGE 65

21. Length of street + length of column = 8700 ft.;

$$\therefore \text{ time} = \frac{8700}{58 \times 2\frac{1}{2}} \text{ min.} = 60 \text{ min.}$$

22. The gallon contains $\frac{277.274}{1728}$ cu. ft.;

$$\therefore \text{ the gallon weighs } \frac{277.274 \times 1000}{1728} \text{ oz.};$$

$$\therefore \text{ the pint weighs } \frac{277274}{16 \times 8 \times 1728} \text{ lb.} = 1.2535 \ldots \text{ lb.}$$

23. Since £$3\frac{9}{10}$ = 1 oz. Troy;

$$\therefore \text{ £}423267 = \frac{423267 \times 1}{3.9} \text{ oz. Troy.}$$

1 oz. Troy = 480 grains;
1 lb. avoir. = 7000 "

24. Distance the train goes in 60×60 sec. $= 20 \times 1760$ yd.;

$$\therefore \text{ " \quad " \quad " } 18 \text{ sec.} = \frac{18 \times 20 \times 1760}{60 \times 60} \text{ yd.}$$
$$= 176 \text{ yd.};$$
$$\therefore \text{ length of bridge} = (176 - 120) \text{ yd.}$$
$$= 56 \text{ yd.}$$

25. $\frac{857142}{999999}=\frac{6}{7}$.

$\frac{6}{7}$ of £10 14*s.* 1*d.* = £9 3*s.* 6*d.*

Again, .85714 of £10 14*s.* 1*d.* = .85714 of 2569*d.*

= 2201.99266*d.*

£9 3*s.* 6*d.* = 2202*d.*

Difference = .00734*d.*, which is less than $\frac{1}{125}$*d.*

26. 1 lb. Troy = $\frac{144}{175}$ lb. avoir.

$$\text{Weight of rings}=\frac{1050\times 28}{12\times 20\times 24}\text{ lb. Troy}$$

$$=\frac{144\times 1050\times 28}{175\times 12\times 20\times 24}\text{ lb. avoir.}=\frac{21}{5}\text{ lb. avoir.}$$

Weight of rings and box = $(\frac{21}{5}+3\frac{1}{2})$ lb. = $7\frac{7}{10}$ lb.

Cost of carrying 1 t. 1 mi. = 5*s.*;

∴ cost of carrying $7\frac{7}{10}$ lb. 144 mi.

$$=\frac{7\frac{7}{10}\times 144\times 5}{2240}\,s.=\frac{99}{40}s.=2s.\ 5\tfrac{7}{10}d.$$

27. Distance walked in 1 sec. = $\frac{3\times 32}{2}$ in.;

∴ " " " $4\frac{1}{2}$ hr. = $\frac{9\times 60\times 60\times 3\times 32}{2\times 2}$ in.;

∴ half " " " 2 " = $\frac{9\times 60\times 60\times 3\times 32}{2\times 2\times 2}$ in.;

∴ " " " " 1 min. = $\frac{9\times 60\times 60\times 3\times 32}{2\times 60\times 2\times 2\times 2}$ in.;

$$\therefore\text{ number of steps taken}=\frac{9\times 60\times 60\times 3\times 32}{36\times 2\times 60\times 2\times 2\times 2}=90.$$

29. 1 oz. Troy per inch = 480 gr. Troy per inch

= $\frac{480}{7000}$ lb. avoir " "

$$=\frac{63360\times 480}{7000}\text{ lb. avoir. per mile}$$

$$=\frac{63360\times 480}{2000\times 7000}\text{ t. per mile}$$

= 2.1723 t. per mile.

Weight required = (2.1723 × .03) t. per mile

= .065169 t. per mile.

31. When *A* is supposed to win he runs 1760 yd. while *B* runs 1749 yd.

Hence, *A*'s rate : *B*'s as 1760 : 1749.
But *A*'s rate is $\frac{4}{3}$ of his actual rate;
$\therefore$ $\frac{4}{3}$ *A*'s actual rate $=1760$,
and *A*'s " " $=\frac{3}{4}$ of $1760=1320$;
$\therefore$ *A*'s rate : *B*'s as 1320 : 1749,
or as 40 : 53.

36. Time in seconds $=\frac{333\times 5280}{66}=26640=7$ hr. 24 min.;

$\therefore$ it will reach Montreal at $(6.25+7.24)$ or 1.49 p.m.

Time the Toronto-train has being going at $8=1$ hr. 35 min.

Distance it goes in 1 hr. 35 min. $=\frac{5700\times 66}{5280}$ min. $=71\frac{1}{4}$ mi.

Distance between Montreal and Toronto-train at 8 a.m. is $(333-71\frac{1}{4})$ mi., or $261\frac{3}{4}$ mi.

Each second they approach $(88+66)$ ft. or 154 ft.

Number of seconds to meet $=\frac{261.75\times 5280}{154}$.

$$\text{Distance gone by Montreal-train}=\frac{261.75\times 5280\times 88}{5280\times 154}\text{ mi.}$$
$$=149\tfrac{4}{7}\text{ mi.}$$

39. The corresponding unit of area is a square each of whose sides is equal to the lineal unit, and the corresponding unit of volume is a cube each of whose edges is equal to the lineal unit.

When the lineal unit is twelve inches, the unit of area is a square each of whose sides is 12 in., or a square whose area is 144 sq. in.; the unit of volume is a cube each of whose edges is 12 in., or a cube whose volume is 1728 cu. in.

43. Number of square inches in 15 sq. ft. $=15\times 144$;

$$\therefore\ \text{pressure}=(15\times 144\times 15)\text{ lb.}$$
$$=16\text{ t. }4\text{ cwt.}$$

When the barometer is at 29, the pressure will evidently be $\frac{1}{30}$ less than before.

$\frac{1}{30}$ of 16 t. 4 cwt. $=10$ cwt. 3 qrs. 5 lb.

47. Number of ounces bought $= 12 \times 16$,

" " sold $= \dfrac{12 \times 7000}{20 \times 24}$.

Cost price $= 12 \times 16 \times 37\tfrac{1}{2}$c.

$= \$72$.

Selling price $= \dfrac{12 \times 7000 \times 40}{20 \times 24}$c.

$= \$70$;

$\therefore$ he loses \$2.

49. It is evident the weight must be a common measure of 8 lb. 20 gr. and 8 lb. 11 oz. 16 dwt. 16 gr.

8 lb. 20 gr. = 56020 gr.

8 lb. 11 oz. 16 dwt. 16 gr. = 51760 gr.

The H. C. F. of 56020 gr. and 51760 gr. is 20 gr.

EXERCISE XX. PAGE 80

15. 35 yd. = 32 m.;

$\therefore$ $69\tfrac{1}{2}$ mi. $= \dfrac{69\frac{1}{2} \times 8 \times 40 \times 5\frac{1}{2} \times 32}{35}$ m.

$= 111835\tfrac{3}{7}$ m.

16. 1 mi. $= (1760 \times 36)$ in. $= \dfrac{63360}{39.371}$ m.

$= 1609.306 \ldots$ metres.

17. $\dfrac{5 \times 5280}{8000}$ ft. = 1 m.

$\dfrac{(5 \times 5280)^3}{8000^3}$ cu. ft. = 1 cu. m.

$\dfrac{216 \times 5^3 \times 5280^3}{8000^3}$ cu. ft. = 216 cu. m.;

$\therefore$ $\dfrac{6 \times 2240 \times 5^3 \times 5280^3}{8000^3}$ lb. = 216000 Kg.

18. Cost of 1927 Ha. $= £\dfrac{10100000}{25.25} = £400000$;

$\therefore$ " " .40467 Ha. $= £\dfrac{(.40467 \times 400000)}{1927} = £84$.

19. Weight of $(3.937)^3$ cu. in. of water $= 1$ Kg.

" " 1 " " " " $= 252.45$ gr.;

$\therefore$ " " $(3.937)^3$ " " " " $= \dfrac{(3.937)^3 \times 252.45}{7000}$ lb.

$= 2.20$ lb.;

$\therefore$ 1 Kg. $= 2.20$ lb.

21. A hectare is a square hectometre, and 100 sq. hectometres make 1 sq. Km.

7 Ha. 25 a. 8 ca. $= 7.2508$ sq. Hm. $= .072508$ sq. Km.

22. Since $2\frac{1}{5}$ lb. $= 1$ Kg.

$$10 \text{ lb.} = \frac{10 \times 1}{2\frac{1}{5}} \text{ Kg.} = 4.\dot{5}\dot{4} \text{ Kg.}$$

$= 4545$ g. $= 4545$ cu. cm of water.

Hence 1 gal. contains 4545 cu. cm.

23.
$$18.60 \text{ fr.} = \$\frac{18.60}{5\frac{1}{6}} = \$\frac{18.60 \times 6}{31}.$$

Cost of 39.37 in. $= \$\dfrac{18.60 \times 6}{31}$;

$\therefore$ " " 1 yd. $= \$\dfrac{36 \times 18.60 \times 6}{39.37 \times 31} = \$3.29$.

24. Volume of bar $= (10 \times 3 \times 2.5)$ cu. cm.

Weight of 1 cu. cm. of water $= 1$ g.;

$\therefore$ " " 1 " " " gold $= 19.36 \times 1$ g.;

$\therefore$ " $(10 \times 3 \times 2.5)$ cu. cm. ... $= \dfrac{10 \times 3 \times 2.5 \times 19.36}{1000}$ Kg.

$= 1.452$ Kg.

25. Circumference of earth $= 40000000 \times 39.37$ in.

$= \dfrac{40000000 \times 39.37}{5280 \times 12}$ mi.;

$\therefore$ diameter of earth $= \frac{113}{355}$ of $\dfrac{40000000 \times 39.37}{5280 \times 12}$ mi.

$= 7911.5 \ldots$ mi.

26. Weight of 1 l. of water $= 1$ Kg.,

$\therefore$ " " 1 l. of mercury $= 13.5 \times 1$ Kg. $= 13.5$ Kg.

27. A stere = 1 cu. m.,

$$\therefore \text{ depth of water} = \frac{2}{3 \times 1.5} \text{ m.} = \tfrac{4}{9} \text{ m.}$$

29. 1 l. = 1 cu. dcm. $= \frac{1}{1000}$ of $(39.37)^3$ cu. in.

$= \frac{1}{1728}$ of $\frac{1}{1000}$ of $(39.37)^3$ cu. ft.

= .0353 … cu. ft.

30. 1 Ha. = 10000 sq. m. = 1000000 sq. dm.

Number of litres = 1000000 × .1 = 100000.

36. Weight of 1 stere of water = 1000 Kg.;

$\therefore$ " " $\frac{11}{10}$ " " ice = 1000 Kg.;

$\therefore$ " " 1 " " " $= \frac{10}{11}$ of 1000 Kg.

$= 909.\dot{0}\dot{9}$ Kg.

39. A litre = 1 cu. dcm. $= (3.937)^3$ cu. in.

Hence, number of litres in a cubic foot $= \dfrac{1728}{(3.937)^3} = 28.31 \ldots$ l.

40. A litre of water weighs 1000 gr.;

$\therefore$ equal volumes of water and sulphuric acid are as 1000 to 1840.

43. Capacity of bin = (8.4 × 5.5 × 4.75) cu. m.

= 219.450 cu. m. = 219450 cu. dcm.

= 219450 l. = 2194.50 Hl.

Exercise XXI. Page 87

25. Since 75° is the central meridian of eastern time, mean solar time at Hamilton must be slower than eastern time by the difference which 4° 54′ make.

4° 54′ correspond to a difference of 19 min. 36 sec.

26. For a reason similar to that assigned in the last question, Quebec mean solar time is faster than standard eastern time by the difference which 3° 47′ make.

27. 42 min. 20 sec. correspond to 10° 35′.

As the time at Banff is slower than standard mountain time, the longitude of Banff must be 105° + 10° 35′, or 115° 35′ west longitude.

28. (*a*) Since 9000 mi. = 360°,

∴ 500 mi. = $\frac{500}{9000}$ of 360° = 20°.

The difference of time is, hence, 20 × 4 min., or 80 min.

EXERCISE XXII. PAGE 92

15. Since 27 men in 14 da. can do a work, working 10 hr. per day,

∴ 27 " 45 " " " $\frac{14}{45}$ of 10 hr. "

and 12 " 45 " " working $\frac{27}{12}$ of $\frac{14}{45}$ of 10 hr. "

or 7 hr. per day.

27. Value of $\frac{2}{7}$ of $\frac{5}{16}$ of $\frac{3}{14}$ of estate = \$603.125;

∴ " $\frac{1}{5}$ of $\frac{3}{16}$ of estate $= \$\dfrac{\frac{1}{5} \text{ of } \frac{3}{16} \times 603.125}{\frac{2}{7} \text{ of } \frac{5}{16} \text{ of } \frac{3}{14}}$

= \$1182.125.

28. Distance 15.5 cwt. is carried = 60 mi.

∴ " 3.25 " " $= \dfrac{15.5 \times 60}{3.25}$ mi. = $286\frac{2}{13}$ mi.

32. Area of walls = $\{(2 \times 6.5 + 2 \times 4.6) \times 3.2\}$ sq. m.

Area of a roll = $(10 \times .5)$ sq. m.

Number of rolls $= \dfrac{22.2 \times 3.2}{10 \times .5} = 14.208.$

33. Number of ares sold = .7 of 605.09 a.

= 423.563 a.

Sum realized = 423.563 × \$380

= \$160953.94.

Number of ares unsold = .3 of 605.09 a.

= 181.527 a.

47. Number supplied for \$1.20 for 50 hr. = 5;

∴ " " " \$21.60 for 60 hr. $= \dfrac{21.60 \times 50 \times 5}{1.20 \times 60} = 75.$

51. Time required by 5 men to reap $12\frac{1}{2}$ ac. = 56 hr.;

∴ " " " 7 " " 15 ac.

$= \dfrac{5 \times 15 \times 56}{7 \times 12\frac{1}{2}}$ hr.

= 48 hr. = 4 da.

56. Time 300 bbl. supply $(3 \times 44 \times 30)$ rounds $= 5$ da.;
∴ " 400 " " $(5 \times 66 \times 40)$ "

$$= \frac{400 \times 3 \times 44 \times 30 \times 5}{300 \times 5 \times 66 \times 40} \text{ da.} = 2 \text{ da.}$$

61. In 1 da., *A* and *B* reap $\frac{1}{3}$ of field;
" 1 " *A* " *C* " $\frac{2}{7}$ " "
and " 1 " *B* " *C* " $\frac{1}{4}$ " "
∴ twice (*A*'s work + *B*'s work + *C*'s work) daily

$$= \frac{1}{3} + \frac{2}{7} + \frac{1}{4} = \frac{73}{84};$$

∴ *A*, *B* and *C* do $\frac{73}{168}$ of the work daily.
Time required to do $\frac{73}{168}$ of work $= 1$ da.;
∴ " " " all the work $= \frac{168 \times 1}{73}$ da. $= 2\frac{22}{73}$ da.

62. *A* and *B* can do $\frac{1}{4}$ of work in 1 hr.,
A " *C* " $\frac{5}{18}$ " " 1 hr.;
∴ two men of *A*'s strength, aided by *B* and *C*, can do $(\frac{1}{4} + \frac{5}{18})$ of work in 1 hr.

Now *B* and *C* can do $\frac{7}{36}$ of work in 1 hr.;
∴ two men of *A*'s strength can do $(\frac{1}{4} + \frac{5}{18} - \frac{7}{36})$ of work in 1 hr.;

∴ *A* can do $\frac{1}{6}$ of work in 1 hr.,
and *A* can do the entire work in 6 hr.

64. One pipe fills $\frac{1}{40}$ of the bath in 1 min.
The other empties $\frac{1}{60}$ of the bath in 1 min.;
∴ when both are running $(\frac{1}{40} - \frac{1}{60})$ or $\frac{1}{120}$ of bath is filled in 1 min.;

∴ the bath is filled in 120 min.

66. Part done by *A* in 1 da. $= \frac{1}{14}$ of $\frac{7}{10}$ of work;
∴ " " " 2 da. $= 2 \times \frac{1}{14}$ of $\frac{7}{10}$ of work
$= \frac{1}{10}$ of work.
Part done by *B* in 2 da. $= 1 - (\frac{7}{10} + \frac{1}{10}) = \frac{1}{5}$;
∴ " " " 1 da. $= \frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$;
∴ *B* would do the whole work in 10 da.

67. In 1 hr. *A* does $\frac{1}{3}$ of the work;
In 1 hr. *B* and *C* do $\frac{2}{3}$ " " "
" *A* " *C* " $\frac{3}{4}$ " " "
Part done by *C* in 1 hr. $= \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$;
$\therefore$ " " *B* " $= \frac{2}{3} - \frac{5}{12} = \frac{3}{12}$.
Time *B* requires to do $\frac{3}{12}$ of work = 1 hr.;
$\therefore$ " *B* " " all the work $= \frac{12 \times 1}{3}$ hr. = 4 hr.

70. In 1 da. *A* does $\frac{1}{3}$ of work;
" $\frac{1}{2}$ " *B* " $\frac{1}{8}$ "

If *A* and *B* had worked as long as *C*, the three would have done $(1 + \frac{1}{3} + \frac{1}{8})$ of work.

But they all do $(\frac{1}{3} + \frac{1}{4} + \frac{1}{5})$ of work in one day;

$$\therefore \text{ number of days } C \text{ worked} = \frac{1 + \frac{1}{3} + \frac{1}{8}}{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}}$$
$$= 1\tfrac{81}{94} \text{ da.};$$

$\therefore$ *A* worked $(1\frac{81}{94} - 1)$ da., or $\frac{81}{94}$ da.
and *B* " $(1\frac{81}{94} - \frac{1}{2})$ " $1\frac{17}{47}$ da.

71. 5 men, 4 boys, and 3 girls do $\frac{1}{9}$ in 1 da. *A*.
3 " 8 " " 6 " $\frac{1}{8}$ " *B*.
4 " 10 " " 9 " $\frac{1}{6}$ " *C*.
Hence, 10 " 8 " " 6 " $\frac{2}{9}$ " 2 *A*.
And 3 " 8 " " 6 " $\frac{1}{8}$ " *B*.
$\therefore$ 7 men do $(\frac{2}{9} - \frac{1}{8})$ "
or $\frac{7}{72}$ "
$\therefore$ 1 man does $\frac{1}{72}$ "
Again, 15 men, 12 boys, and 9 girls do $\frac{3}{9}$ " 3 *A*.
And 4 " 10 " " 9 " $\frac{1}{6}$ " *C*.
$\therefore$ 11 men " 2 boys do $\frac{3}{9} - \frac{1}{6}$ in 1 da.
$\therefore$ 2 " $\frac{3}{9} - \frac{1}{6} - \frac{11}{72}$ "
or $\frac{1}{72}$ "
$\therefore$ 1 boy does $\frac{1}{144}$ "
But 5 men, 4 boys, and 3 girls do $\frac{1}{9}$ "
$\therefore$ 3 " $\frac{1}{9} - \frac{5}{72} - \frac{4}{144}$ "
$\therefore$ 1 girl does $\frac{1}{216}$ "

Hence, 3 men, 5 boys, and 4 girls do $\frac{3}{72}+\frac{5}{144}+\frac{4}{216}$ in 1 da.
or $\frac{41}{432}$; "
∴ time to do all the work is $\frac{432}{41}$ da.
or $10\frac{22}{41}$ da.

76. Part done by *A*, 2 *B*'s, and *C* in 1 da. $=\frac{1}{8}+\frac{1}{12}$.
" " *A*, *B*, and *C* " $=\frac{1}{6}$;
∴ " " 2 *A*'s, 2 *B*'s, 2 *C*'s " $=\frac{1}{3}$;
∴ " " *A* and *C* " $=\frac{1}{3}-(\frac{1}{8}+\frac{1}{12})$
$=\frac{1}{8}$;
∴ time required by *A* and *C* = 8 da.

78. 30 men and 10 boys reap 130 ac. in 4 da.
14 " " 10 " " 66 " 4 da.;
∴ 16 " reap 64 ac. in 4 da.;
∴ 1 man reaps 1 " 1 "
But 6 men and 2 boys reap 13 " 2 da.;
∴ 2 boys reap 1 ac. in 2 da.;
∴ 1 boy reaps $\frac{1}{4}$ " 1 "
and 2 men and 2 boys reap $2\frac{1}{2}$ " 1 da.
" " " 10 " $\frac{10\times 1}{2\frac{1}{2}}$ da = 4 da.

79. Time *A* walks 20 mi. $=20\times 11$ min. $=220$ min.
" " " $=(220+45)$ min. $=265$ min.;
∴ *B*'s rate $=\frac{265}{20}$ min. per mile
$=13\frac{1}{4}$ " "
Again, time *A* walks 5 mi. $=55$ "
" *B* " " $=66\frac{1}{4}$ "
∴ *A* wins by $11\frac{1}{4}$ min.;
∴ distance *B* walks in $11\frac{1}{4}$ min. $=\frac{11\frac{1}{4}}{13\frac{1}{4}}$ mi. $=\frac{45}{53}$ mi.

80. Since *A* can make 50 when *B* makes 45,
and *A* " " 50 " *C* " 40;
∴ *B* " " 90 " *C* " 80;
∴ *B* can give *C* 10 points.

82. Let 1 represent the quantity of water in each cistern; then, quantity of water which runs out of first cistern in 1 hr. $= \frac{1}{5}$;

∴ " " " " in number of hours required $= \frac{\text{number of hours}}{5}$.

" " " " second cistern " $= \frac{\text{number of hours}}{4}$.

By conditions of question,

$$1 - \frac{\text{number of hours}}{5} = 2 \times \left(1 - \frac{\text{number of hours}}{4}\right)$$

$$= 2 - \frac{\text{number of hours}}{2};$$

$$\therefore \frac{3 \times \text{number of hours}}{10} = 1,$$

and number of hours $= \frac{10}{3}$

$= 3\frac{1}{3}$.

84. Distance *A* runs in 1 min. $= (2\frac{1}{3} \div 16\frac{4}{5})$ mi.

$= (\frac{7}{3} \times \frac{5}{84})$ mi.;

∴ " " 34 " $= (34 \times \frac{7}{3} \times \frac{5}{84})$ mi.,

and " *B* " 34 " $= \frac{18}{17} \times 34 \times \frac{7}{3} \times \frac{5}{84}$

$= 5$ mi.;

∴ length of course $= (2\frac{1}{3} + 5)$ mi.

$= 7\frac{1}{3}$ mi.

85. Time to go 1 mi. out $= \frac{1}{10}$ hr.

Time to come 1 mi. back $= \frac{1}{5}$ hr.;

∴ time to go 1 mi. out and back $= (\frac{1}{10} + \frac{1}{5})$ hr. $= \frac{3}{10}$ hr.

Distance gone in $\frac{3}{10}$ hr. $= 1$ mi.;

∴ " " " 5 hr. $= (5 \times \frac{10}{3})$ mi.

$= 16\frac{2}{3}$ mi.

87. *B* walks at the rate of $\frac{50}{12}$ mi., or $4\frac{1}{6}$ mi. per hour.

∴ *B* " 20 mi. in $(20 \div 4\frac{1}{6})$ hr. $= 4\frac{4}{5}$ hr.

∴ *A* " 20 " 1 hr. $+ 4\frac{4}{5}$ hr. $= 5\frac{4}{5}$;

∴ *A* " 50 " $\frac{50 \times 5\frac{4}{5}}{20}$ hr. $= 14\frac{1}{2}$ hr.;

∴ *A* reaches London at 6 hr. 30 min. p.m.

88. 9 boys are equivalent to 6 men;
since 12 men do $\frac{3}{4}$ of the work in $6\frac{1}{2}$ hr.,

$\therefore$ 1 man does $\frac{1}{4}$ " " " $\frac{12\times 13}{3\times 2}$ hr.;

$\therefore$ 17 men do $\frac{1}{4}$ " " " $\frac{12\times 13}{17\times 3\times 2}$ hr., or $1\frac{9}{17}$ hr.

89. Part done by 2 men and 4 boys hourly $=\frac{1}{2}$.
" " " 2 " " 1 boy " $=\frac{1}{3}$;
$\therefore$ part done by 3 boys " $=\frac{1}{2}-\frac{1}{3}$
$=\frac{1}{6}$;
$\therefore$ " " " 1 boy " $=\frac{1}{18}$;
hence, 1 boy would do the whole in 18 hr.
Part done by 2 men hourly $=\frac{1}{3}-\frac{1}{18}$
$=\frac{5}{18}$;
$\therefore$ " " " 1 man " $=\frac{5}{36}$;
hence, 1 man would do the whole in $\frac{36}{5}$ hr., or $7\frac{1}{5}$ hr.
Part done by 1 man and 1 boy hourly $=\frac{5}{36}+\frac{1}{18}$
$=\frac{7}{36}$;
hence, 1 man and 1 boy would do the whole in $\frac{36}{7}$ hr., or $5\frac{1}{7}$ hr.

90. Time required for 15 men working 9 hr. a day to finish the work $=16$ da.;

$\therefore$ to do the work in 1 da. 1 man must work $(15\times 16\times 9)$ hr.;

$\therefore$ to do it in 12 da. 18 men must work $\frac{15\times 16\times 9}{12\times 18}$ hr., or 10 hr.

92. 5 men in $\frac{53}{25}$ hr. do $\frac{901}{1500}$ of work;
$\therefore$ 1 man " 1 " does $\frac{17}{300}$ "
$\therefore$ 3 men " 3 " do $\frac{51}{100}$ "
hence, 7 boys " 3 " " $\frac{49}{100}$ "
1 boy " 1 " does $\frac{7}{300}$ "
$\therefore$ 6 boys " 1 " do $\frac{7}{50}$ "
Work to be done by 6 boys $=1-\frac{901}{1500}=\frac{599}{1500}$.
Time for 6 boys to do $\frac{7}{50}$ $=1$ hr.;
$\therefore$ " " " $\frac{599}{1500}=\frac{\frac{599}{1500}\times 1}{\frac{7}{50}}$ hr. $=2.8\dot{5}2380\dot{9}$ hr.

93. *A* does $\frac{1}{24}$ of work daily;
∴ he " $\frac{1}{48}$ " in half a day;
∴ in two days *A* and *B* do $(1 - \frac{5}{6} - \frac{1}{48})$ of work;
∴ " one day *A* " *B* " $\frac{7}{48} \div 2 = \frac{7}{96}$ "
∴ " " " *B* does $\frac{7}{96} - \frac{1}{24} = \frac{3}{96}$ "
∴ *B* does the work in $\frac{96}{3}$ da., or 32 da.

94. 6 men reap 35 ac. in 7 × 12 hr.;

∴ 1 man reaps 1 " $\dfrac{6 \times 7 \times 12}{35}$ hr.;

∴ 9 men reap 45 " $\dfrac{45 \times 6 \times 7 \times 12}{9 \times 35}$ hr. = 72 hr.;

∴ they will take 9 da. of 8 hr. each.

98. If *A* works 10 da., *B* can finish in 2 da.
If *A* " 2 " *B* " " " 8 da.;
∴ *A* must do in 8 da. as much as *B* in 6 da.;
∴ *A* " " 10 " " *B* " $7\frac{1}{2}$ da.;
∴ *B* does the work in $9\frac{1}{2}$ da.
Again, *B* " as much in 2 da. as *A* in $\frac{8}{6}$ of 2 da.;
∴ *A* " the work in $12\frac{2}{3}$ da.

99. Amount which the work falls behind daily
$= (\frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{8} - \frac{1}{10})$ of a day's work;
∴ in 84 da. it falls behind $84 \times (\frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{8} - \frac{1}{10})$ of a day's work = 17.6 da. work;
∴ 17 men must do .6 of a day's work more than average;

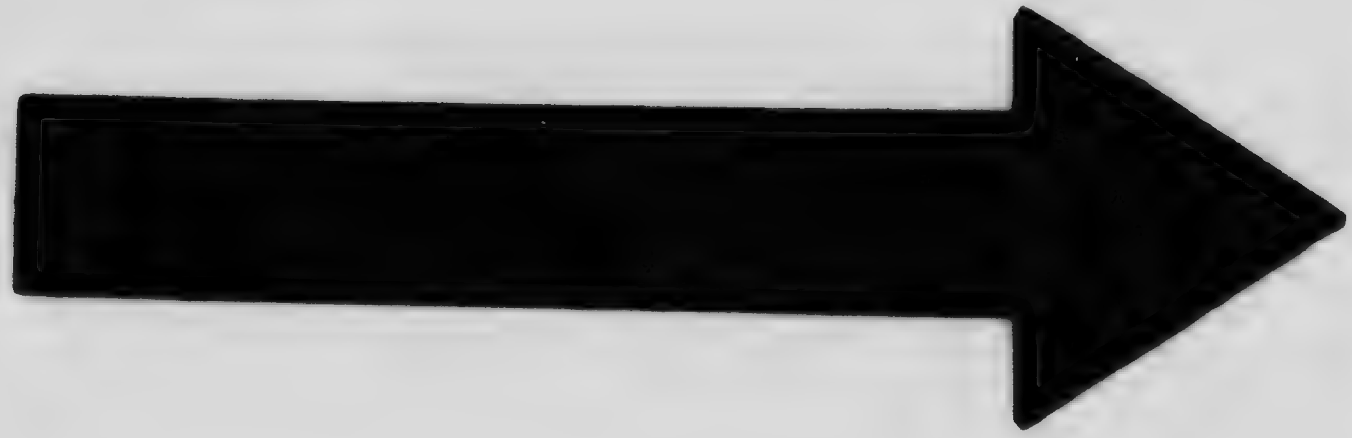
∴ 1 man " $\dfrac{.6}{17}$ " " " "

$= \frac{3}{85}$ " " " "

102. From midnight on Sunday to 6 p.m. on Wednesday is 66 hr.

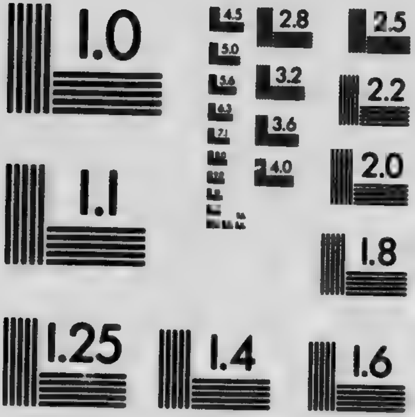
Time lost by the clock in 66 hr. $= \dfrac{66 \times 4}{12}$ min. = 22 min.;

∴ taking away the 10 min. already gained, the clock will indicate 12 min. to 6, or 5 hr. 48 min.



MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



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103. Sum of rates per second $= \dfrac{92+84}{1\frac{1}{2}}$ ft. $= \frac{352}{3}$ ft.

Difference " " " $= \dfrac{92+84}{6}$ ft. $= \frac{88}{3}$ ft.;

$\therefore$ twice the rate of faster " $= \dfrac{352+88}{3}$ ft.;

$\therefore$ " " " hour $= \dfrac{60 \times 60 \times 440}{2 \times 5280 \times 3}$ mi.

$= 50$ mi.

Again, twice the rate of the slower per second

$= \dfrac{352-88}{3}$ ft. $= \frac{264}{3}$ ft.;

$\therefore$ rate of slower per hour $= \dfrac{60 \times 60 \times 264}{2 \times 5280 \times 3}$ mi.

$= 30$ mi.

104. Distance A goes in 10 sec. $= \dfrac{10 \times 4200}{60 \times 60}$ m.

$= \frac{70}{6}$ m.;

$\therefore$ distance train goes in 10 sec. $= (88 + \frac{70}{6})$ m.

$= \frac{598}{6}$ m.;

$\therefore$ " " " 9 " $= \frac{9}{10}$ of $\frac{598}{6}$ m.

$= \frac{897}{10}$ m.;

$\therefore$ " B " 9 " $= (\frac{897}{10} - 88)$ m.

$= \frac{17}{10}$ m.;

$\therefore$ B's rate in metres per hour $= \dfrac{60 \times 60 \times 17}{9 \times 10}$ m. $= 680$ m.

106. Since 5 horses $= 84$ sheep;

$\therefore$ " 10 " $= 168$ "

$\therefore$ 10 horses and 132 sheep $= (168 + 132)$ "

$= 300$ "

And 15 horses and 148 " $= (252 + 148)$ "

$= 400$ "

Cost of keeping 300 " $= \$202$;

$\therefore$ " " 400 " $= \$\dfrac{400 \times 202}{300} = \$269\frac{1}{3}$.

110. Distance the 1st man goes in 15 sec. $= \dfrac{15 \times 3200}{60 \times 60}$ m.

$= \frac{40}{3}$ m.

" " train " 15 " $= (105.6 + \frac{40}{3})$ m.

$= \dfrac{356.8}{3}$ m.;

∴ " " " " 12 " $= \frac{12}{15}$ of $\dfrac{356.8}{3}$ m.

$= \dfrac{1427 \cdot 2}{15}$ m.;

∴ " " 2nd " " 12 " $= \left(105.6 - \dfrac{1427.2}{15}\right)$ m.

$= \dfrac{156.8}{15}$ m.;

∴ rate of 2nd man per hour $= \dfrac{60 \times 60 \times 156.8}{12 \times 15}$ m.

$= 3136$ m.

111. *A*'s wages for $12\frac{6}{7}$ da. $=$ *A*'s wages for $7\frac{1}{2}$ da. $+$ *B*'s wages for $7\frac{1}{2}$ da.;

∴ *A*'s wages for $(12\frac{6}{7} - 7\frac{1}{2})$ da. $=$ *B*'s " " $7\frac{1}{2}$ da.;

∴ *A*'s " " $12\frac{6}{7}$ da. $=$ *B*'s " " $\dfrac{12\frac{6}{7} \times 7\frac{1}{2}}{5\frac{5}{14}}$ da.

$= 18$ da.

113. The first strikes the 7th stroke after $\dfrac{6 \times 35}{11}$ sec.;

" second " " " " $\dfrac{6 \times 25}{11}$ sec.;

∴ the difference $= \dfrac{6 \times 10}{11}$ sec. $= \frac{1}{11}$ min.

117. Since the fast train goes as far in 5 hr. as the slow one does in 6 hr., the rates are as 5 : 6.

Since the fast train gains 10 mi. in 2 hr., it gains 5 mi. in 1 hr.

Hence, $\frac{6}{5}$ of rate of slow train = rate of slow train + 5 mi.;

∴ $\frac{1}{5}$ " " " " " $= 5$ mi.,

and " " " " $= 5 \times 5$ mi. $= 25$ mi.

Rate of fast train $= (25 + 5)$ mi. $= 30$ mi.

118. Time for $(9+2\times 12+3\times 7)$ boys $= 250$ da.;

∴ " " $(18+2\times 15+3\times 9)$ "

$$= \frac{(9+2\times 12+3\times 7)\times 250}{18+2\times 15+3\times 9} \text{ da.}$$

$= 180$ da.;

∴ to do double the work they would be 2×180 da., or 360 da.

121. Since time past noon $= \frac{4}{5}$ time to midnight;

∴ $\frac{5}{4}$ of " " " = " "

∴ time past noon $+ \frac{5}{4}$ " " " " $= 12$ hr.

$\frac{9}{4}$ " " " " $= 12$ hr.;

∴ " " " $= \frac{4}{9}$ of 12 hr.

$= 5$ hr. 20 min.

Hence, the time is 20 min. past 5.

122. Distance the train goes in 10 sec. $= \frac{10\times 48000}{60\times 60}$ m.

$= 133\frac{1}{3}$ m.

" " man " " $= \frac{10\times 4800}{60\times 60}$ m.

$= 13\frac{1}{3}$ m.

Since the distance the train goes in 10 sec. is the sum of the length of the train and the distance walked by the man in this time;

∴ the length of the train $= (133\frac{1}{3} - 13\frac{1}{3})$ m.

$= 120$ m.

125. Area of field $= 57600$ sq. m.

Length of shorter side $= (\sqrt{28800})$ m.

$= 169.7 \ldots$ m.

133. Weight of 1 Hl. of water $= 100$ Kg.;

∴ " " " sulphuric acid $= 1.84\times 100$ Kg.

$= 184$ Kg.

134. If the fraction is a proper fraction, its value is increased by adding the same number to both terms. If it is an improper fraction, its value is diminished.

136. Let a child get 1 share and 8 children 8 shares,
then a woman gets 2 shares and 6 women 12 shares;
and a man gets 6 shares and 5 men 30 shares.
Hence, 50 children's shares = \$379.50,
and a child's share = \$7.59;
a woman's " = \$15.18;
a man's " = \$45.54.
Hence, the men get 5 × \$45.54, or \$227.70;
the women " 6 × \$15.18, or \$91.08;
the children " 8 × \$7.59, or \$60.72.

137. Cost of 1000 bu. at 74c. = \$740.
This is less than the actual cost by \$19.
Each bushel of the dearer kind increases the cost by (82 − 74)c., or 8c.;
∴ number of bushels of dearer kind is $\frac{1900}{8}$ or $237\frac{1}{2}$.

140. The watch goes 1430 min. in 1440 min. of time;
∴ 1430 min. on the watch = 1440 min.;
∴ 1440 " " " $= \frac{1440 \times 1440}{1430}$
$= 24$ hr. $10\frac{10}{143}$ min.;
∴ time is 12 hr. $10\frac{10}{143}$ min.

141. If all were satin the cost would be (24 × 20)c. more.
Hence, cost of 40 yd. satin = \$(49.20 + 4.80) = \$54;
∴ " 1 " " $= \$\frac{54}{40} = \$1.35$,
and " 1 " silk = \$1.15.

143. Time to go 1 mi. out $= \frac{1}{10}$ hr.
Time to come 1 mi. back $= \frac{2}{7}$ hr.;
∴ time to go 1 mi. and come " $= (\frac{1}{10} + \frac{2}{7})$ hr.
$= \frac{27}{70}$ hr.
Distance he goes out and comes back in $\frac{27}{70}$ hr. = 1 mi.
" " " " " $4\frac{2}{3}$ hr. $= \frac{4\frac{2}{3} \times 1}{\frac{27}{70}}$ mi.
$= 12\frac{8}{81}$ mi.

145. Since 1 sq. in. = 6.4813669 sq. cm.,

∴ the pressure on 6.4813669 sq. cm. = (7000 × 14.7) grains.

But 15.432 grains = 1 gram;

$$\therefore (7000 \times 14.7) \text{ grains} = \frac{7000 \times 14.7}{15.432} \text{ grams,}$$

$$\text{and the pressure on 1 sq. cm.} = \frac{7000 \times 14.7}{6.4813669 \times 15.432} \text{ grams}$$
$$= 1028.78 \text{ gr.}$$

147. Rate down stream = 12 mi. per hour.

" up " = 9 " "

∴ distance gone up and back in $(\frac{1}{9} + \frac{1}{12})$ hr. = 1 mi.;

$$\therefore \text{ " " " " } 21 \text{ hr.} = \frac{21}{\frac{1}{9} + \frac{1}{12}} \text{ mi.}$$
$$= 108 \text{ mi.}$$

The cities are, hence, 108 mi. apart.

151. Area of roadway = (330 × 22) sq. yd.

Value of 1 ac., or 4840 sq. yd. = $84.

$$\text{" " } (330 \times 22) \text{ " "} = \$\frac{330 \times 22 \times 84}{4840}$$
$$= \$126.$$

153. Time to go the journey by express $= \frac{7}{4}$ hr.

" " " " accommodation $= \frac{35}{12}$ hr.

Difference in time $= (\frac{35}{12} - \frac{7}{4})$ hr.

$= \frac{14}{12}$ hr.

Value of $\frac{14}{12}$ hr. = 65c.

" " 1 hr. $= \frac{12}{14}$ of 65c.

$= 55\frac{5}{7}$c.

154. Work done by $A = \frac{1}{3}$.

" " " $B = \frac{5}{9}$ of $\frac{2}{3}$.

" " " $C = \frac{5}{12}$ of $\frac{4}{9}$ of $\frac{2}{3}$.

" " " $D = \frac{7}{12}$ of $\frac{4}{9}$ of $\frac{2}{3}$

$= \frac{14}{81}$.

Sum received by A for $\frac{1}{3}$ work = $27;

∴ " " " D " $\frac{14}{81}$ " $= \frac{14}{27}$ of \$27

= $14.

155. Value of 1 lb. rice $= 5$c.;

$\therefore$ " " 12 " " $= 12 \times 5$c.;

$\therefore$ " " $8\frac{1}{2}$ " sugar $= 12 \times 5$c.;

$\therefore$ " " $12\frac{1}{2}$ " " $= \dfrac{12\frac{1}{2} \times 12 \times 5}{8\frac{1}{2}}$c.;

$\therefore$ " " $2\frac{1}{4}$ " tea $= \dfrac{12\frac{1}{2} \times 12 \times 5}{8\frac{1}{2}}$c.;

$\therefore$ " " $1\frac{1}{4}$ " " $= \dfrac{1\frac{1}{4} \times 12\frac{1}{2} \times 12 \times 5}{2\frac{1}{4} \times 8\frac{1}{2}}$c.;

$\therefore$ " " $3\frac{3}{4}$ " coffee $= \dfrac{1\frac{1}{4} \times 12\frac{1}{2} \times 12 \times 5}{2\frac{1}{4} \times 8\frac{1}{2}}$c.;

$\therefore$ " " 1 " " $= \dfrac{1\frac{1}{4} \times 12\frac{1}{2} \times 12 \times 5}{3\frac{3}{4} \times 2\frac{1}{4} \times 8\frac{1}{2}}$c. $= 13\frac{11}{153}$c.

156. Weight of a cubic inch of water $= \dfrac{62\frac{1}{2}}{1728}$ lb. $= \frac{125}{3456}$ lb.

" " gallon of " $= \dfrac{277.274 \times 125}{3456}$ lb.

" " " " mercury $= \dfrac{13\frac{1}{2} \times 277.274 \times 125}{3456}$ lb.

Hence, number of gallons of mercury to weigh 2000 lb.

$$= 2000 \div \left(\frac{13\frac{1}{2} \times 277.274 \times 125}{3456}\right) = 14.77 \text{ gal.}$$

157. Number of days from 25th Oct. to 16th Feb. $= 115$.
Quantity of oil burned in 1 hr. $= (115 \times 8 \times .5)$ pt.
Hence, number of hours the lamps were lighted is

$$(231\tfrac{1}{2} \times 8) \div (115 \times 8 \times .5), \text{ or } 4\tfrac{3}{115} \text{ hr.}$$

158. His wage is now $\frac{22}{17}$ of what it was;
$\therefore$ former wage was $\frac{17}{22}$ of present wage.
Hence, $\frac{5}{22}$ of his present wage must be taken off.

162. (*a*) Number of cubic feet in the stick $= 24 \times \frac{21}{12} \times \frac{21}{12}$.
Weight of stick $= (24 \times \frac{21}{12} \times \frac{21}{12} \times 24)$ lb. $= 1764$ lb.

(*b*) 62.5 lb. $= 1$ cu. ft. of water;

$\therefore$ 1764 lb. $= \dfrac{1764}{62.5}$ cu. ft. of water $= 28.224$ cu. ft.

163. $\frac{1080}{7\frac{1}{2}}$ pt., or 18 gal., were sold.

Hence, $\frac{4}{7}$ of (contents less 8 gal.) = 18 gal.
Hence, " " " $= \frac{7}{4}$ of 18 gal.
$= 31\frac{1}{2}$ gal.;
∴ contents $= 39\frac{1}{2}$ gal.

164. Perimeter of a quarter-section = 2 mi. = 2 × 5280 ft.
Length of wire for fence = 4 × 2 × 5280 ft.

$$\text{Weight of wire} = \frac{4 \times 2 \times 5280 \times 1}{16} \text{ lb.}$$
$$= 2640 \text{ lb.}$$

165.
$$\text{Length of wall} = \frac{3217.5}{10 \times 2.75} \text{ ft.} = 117 \text{ ft.}$$

By drawing the figure it will be seen that
2 × length + 2(length − 16 ft. − $5\frac{1}{2}$ ft.) = 117 ft.
Hence, length is 40 ft. and breadth 24 ft.

166. Since 180 sq. ft. sell for $35,
∴ 43560 " " " $\$\frac{43560 \times 35}{180}$, or $8470.

167. 4 min. 40 sec. = 280 sec.

If the whole mile were run at 6 yd. per second in 280 sec., the distance run would be 1680 yd., or 80 yd. short of the mile.

Hence, he must increase his rate 80 sec., or 560 yd., before the end of the running.

169. Distance man goes in 10 sec. $= \frac{10 \times 4 \times 1760}{60 \times 60}$ yd. $= \frac{176}{9}$ yd.;
∴ " train " 10 " $= (88 + \frac{176}{9})$ yd. $= \frac{968}{9}$ yd.;
∴ rate of train per hour $= \frac{60 \times 60 \times 968}{10 \times 1760 \times 9}$ mi. = 22 mi.;
∴ distance train goes in 20 min. $= \frac{22}{3}$ mi.,
and " man " 20 min. $= \frac{4}{3}$ mi.;
∴ distance between the two men. $= (\frac{22}{3} - \frac{4}{3})$ mi. = 6 mi.
Again, distance second man goes in 9 sec.
$$= \left(\frac{9 \times 22 \times 1760}{60 \times 60} - 88\right) \text{yd.} = \frac{88}{10} \text{ yd.};$$

$$\therefore \text{ rate of second man per hour} = \frac{60 \times 60 \times 88}{1760 \times 9 \times 10} \text{ mi.} = 2 \text{ mi.}$$

Hence, the first man gains at the rate of 2 mi. per hour.

Time to gain 2 mi. = 1 hr.;

∴ " " " 6 mi. = 3 hr.

In 3 hr. the train goes 3 × 22 mi., or 66 mi.

But in 3 hr. the second man has gone 3 × 2 mi., or 6 mi.;

∴ when the men are together the train is (66 – 6) mi. away.

EXERCISE XXIII. PAGE 111

7. Aggregate price from 1836 to 1842 = 7 × 63*s*. = 441*s*.;

price for 1836 = 62*s*. 3*d*.;

∴ " " " 1837 " 1842 = 378*s*. 9*d*.

" " " 1837 " 1843 = 7 × 64*s*. 2*d*. = 449*s*. 2*d*.;

∴ average price during 1843 = 449*s*. 2*d*. - 378*s*. 5*d*.

= 70*s*. 5*d*.

9. By examining the following table the truth of this statement is readily seen : —

1	2	3	4	5	6	7	8	9	10	11	12
2	[illegible]	[illegible]	8	10	12	14	16	18	20	22	24
[illegible]	[illegible]	9	12	15	18	21	24	27	30	33	36
4	[illegible]	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

12. A's salary $= \frac{4}{3}$ B's salary;
∴ \$1440 $- \frac{4}{3}$ B's “ = A's increase;
∴ $\frac{9}{4} \times$ (\$1440 $- \frac{4}{3}$ B's salary) + B's “ = \$1440;
∴ \$3240 $- 3 \times$ B's salary + B's “ = \$1440;
∴ $2 \times$ B's “ = \$(3240 − 1440);
∴ B's “ = \$$\frac{1800}{2}$ = \$900;
∴ A's “ = $\frac{4}{3}$ of \$900 = \$1200.

13. First lot cost 8c. and second lot 12c. a dozen;
∴ 1 doz. of each will cost 20c., or 2c. less than average.
But the 5 doz. at 12c. cost 5c. more than average;
∴ number of dozen must be sufficient to reduce average 5c.
2c. reduction on 1 doz. of each.
5c. “ “ $2\frac{1}{2}$ “ “

14. Number of boys at end $= \frac{10}{7}$ of number of boys at first.
“ “ girls “ $= \frac{8}{9}$ “ “ “ girls “
$\frac{19}{15}$ of (number of boys at first + number of girls at first)
$= \frac{10}{7}$ “ “ “ “ “ $+ \frac{8}{9}$ of number of girls at first;
∴ $\frac{10}{7}$ “ “ “ “ “ $- \frac{19}{15}$ “ “ boys “
$= \frac{19}{15}$ “ “ “ girls “ $- \frac{8}{9}$ “ “ girls “
∴ $\frac{17}{105}$ “ “ “ boys “ $= \frac{51}{135}$ “ “ “ “
∴ $\frac{1}{7}$ “ “ “ “ “ $= \frac{1}{3}$ “ “ “ “

Hence, there are 7 boys to 3 girls at first.

17. If there is one man receiving \$16 per week, the aggregate wages are \$(16 + 42 + 48 + 50);

∴ the average weekly wage $= \$\,\frac{16+42+48+50}{1+3+4+5} = \$12.$

21 Aggregate weight of 10 boxes = 10 × 36.5 lb. = 365 lb.
“ “ “ 4 “ = 4 × 25.75 lb. = 103 lb.
“ “ “ 4 “ = 4 × 40 lb. = 160 lb.;
∴ “ “ “ 2 “ = (365 − 103 − 160) lb. = 102 lb.,
and average weight of a box $= \frac{102}{2}$ lb. = 51 lb.

25. Average fineness $= \frac{8 \times 21 + 12 \times 22 + 10 \times 18 + 20 \times 16}{8 + 12 + 10 + 20 + 32}$ carats
= 11.36 … carats.

26. Average weight of 3 boys $= \frac{792 - 5 \times 87}{3}$ lb. = 119 lb.

27. Time of running at 65 Km per hour

$= (8 - 3\frac{2}{3} - 2\frac{1}{2})$ hr. $= 1\frac{5}{6}$ hr.

Aggregate distance gone

$= (3\frac{2}{3} \times 48.4 + 2\frac{1}{2} \times 60.8 + 1\frac{5}{6} \times 65)$ Km

= 448.633 ... Km.

Average speed per hour

$= \frac{448.633}{8}$ Km. = 56.079 ...

29. Aggregate height of 6 boys = (6 × 1.72) m. = 10.32 m.
" " " 5 " = (5 × 1.725) m. = 8.625 m.
∴ height of 6th boy = 1.695 m. = 169.5 cm.

Exercise XXIV. Page 117

4. Quantity of butter from $2\frac{1}{2}$ lb. of milk

$= \frac{24}{100}$ of $\frac{18}{100}$ of $2\frac{1}{2}$ lb.;

∴ milk required to produce $\frac{24}{100}$ of $\frac{18}{100}$ of $2\frac{1}{2}$ lb. of butter

= 1 qt.;

∴ " " " " 486 lb. of butter

$= \frac{486 \times 1}{\frac{24}{100} \times \frac{18}{100} \times 2\frac{1}{2}}$ qt.

= 4500 qt.

18. $\frac{1}{3}$ of money withdrawn = $4500;
∴ " " = $13500;
∴ $\frac{5}{8}$ of money he had in bank = $13500;
∴ " " " " " $= \frac{8}{5}$ of $13500
= $21600;
∴ sum still remaining = $(21600 − 13500)
= $8100.

19. 5% of half the number = $2\frac{1}{2}\%$ of the number;
∴ $(3\% + 2\frac{1}{2}\%)$ of " = 5 ;
∴ number $= \frac{100 \times 55}{5\frac{1}{2}} = 1000.$

24. Wine $= \frac{2}{5}$ of mixture $= \frac{3}{8}$ of (mixture + 4 gal.);
$\therefore\ \frac{2}{5}$ " " $= \frac{3}{8}$ " " $+ \frac{3}{8}$ of 4 gal.;
$\therefore\ (\frac{2}{5} - \frac{3}{8})$ " " $= \frac{3}{8}$ " 4 gal.;
$\therefore\ \frac{1}{40}$ " " $= \frac{3}{8}$ " 4 gal.;
$\therefore$ " $= 40 \times \frac{3}{8}$ of 4 gal.
$= 60$ gal.

25. A gain of 20 % on $\frac{3}{8}$ cost $= \frac{3}{40}$ cost.
" " $12\frac{1}{2}\%$ " $\frac{3}{10}$ " $= \frac{3}{80}$ "
A loss " 25 % " $\frac{13}{40}$ " $= \frac{13}{160}$ "
$\therefore$ net gain $= (\frac{3}{40} + \frac{3}{80} - \frac{13}{160})$ cost
$= \frac{5}{160}$ cost;
$\therefore\ \frac{5}{160}$ cost $=$ \$112;
$\therefore$ " $= \frac{160}{5}$ of \$112
$=$ \$3584.

28. 1 per cent on \$5420 gives \$54.20.
Income at the lower rate on \$9970 $=$ \$(453 − 54.20)
$=$ \$398.80;
" " " \$100 $= \$\dfrac{100 \times 398.90}{9970}$, or \$4;
hence the rates are 4% and 5%.

30. 20%, or $\frac{1}{5}$ of the wheat grown in the country
$= 10000000$ qr.;
$\therefore$ wheat grown $= (5 \times 10000000)$ qr.
$= 50000000$ qr.

31. The average dividend $= \$\dfrac{.75 + .60}{2} = \$.67\frac{1}{2}$.

His debts are $\dfrac{100}{67\frac{1}{2}}$ of \$2700 = \$4000.

32. Before selling to *D* he had 579 sheep.
" " " *C* " $\dfrac{100}{96\frac{1}{2}}$ of 579, or 600 sheep.
" " " *B* " 600 + 90, or 690 "
" " " *A* " $\frac{100}{92}$ of 690, or 750 "

33. $\frac{4}{100}$ of $\frac{4}{5}$ of the capital $= \$32000$;

$\therefore$ $\frac{4}{100}$ of the whole " $= \$\frac{5 \times 32000}{4}$,

and " " " $= \$\frac{100 \times 5 \times 32000}{4 \times 4}$

$= \$1000000$.

$\frac{10}{100}$ of $\frac{1}{5}$ of " $= \$20000$;

$\therefore$ $\frac{52}{100}$ of the receipts $= \$(32000 + 20000)$,

and " $= \$\frac{100 \times 52000}{52}$

$= \$100000$.

34. Capital at end of first year $= \frac{120}{100}$ of original capital.

" " " second " $= \frac{137\frac{1}{2}}{100}$ of $\frac{120}{100}$ of "

" " " third " $= \frac{60}{100}$ of $\frac{137\frac{1}{2}}{100}$ of $\frac{120}{100}$ of capital

$= \frac{99}{100}$ of original capital;

$\therefore$ original capital $-$ $\frac{99}{100}$ of original capital $= \$200$;

$\therefore$ original capital $= 100 \times \$200 = \$20000$.

37. In the last 4 yr. he saves £200 + £120, = £320;

$\therefore$ his income $-$ $\frac{9}{10}$ (his income + £40) = £80;

$\therefore$ $\frac{1}{10}$ of " " $-$ £36 = £80;

$\therefore$ his income = 10 × (£36 + £80)

= £1160.

38. Quantity of cloth required $= (11\frac{1}{2} \times 1\frac{1}{4})$ sq. yd.

Width after shrinking $= (\frac{90}{100}$ of $1\frac{1}{4})$ yd.;

$\therefore$ measure of length required if there is no shrinking

$$= \frac{11\frac{1}{2} \times 1\frac{1}{4}}{\frac{90}{100} \text{ of } 1\frac{1}{4}}.$$

As the shrinkage amounts to $\frac{1}{8}$ of the length bought,

$\therefore$ $\frac{7}{8}$ of length bought $= \frac{11\frac{1}{2} \times 1\frac{1}{4}}{\frac{90}{100} \text{ of } 1\frac{1}{4}}$ yd.;

$\therefore$ " " $= \frac{8 \times 11\frac{1}{2} \times 1\frac{1}{4}}{7 \times \frac{90}{100} \times 1\frac{1}{4}}$ yd. $= 14\frac{38}{63}$ yd.

39. Money + value of purse = \$82;

∴ 3 × money + 3 × " " = \$246;

and $\frac{85}{100}$ of " + 3 × " " = \$74;

$\frac{215}{100}$ of money = \$172;

∴ " = $\frac{100}{215}$ of \$172 = \$80;

∴ value of purse = \$(82 − 80) = \$2.

40. Every 100 male and female criminals increased to 101.8; but every 100 males decreased to 100 − 4.6 = 95.4, that is 101.8 − 95.4 = 6.4 below the percentage average. And every 100 females increased to 109.8, that is 8 above the percentage average. In order to make the increase counterbalance the decrease, we must have more males than females in the ratio of the difference just found; that is males : females :: 8 : 6.4 = 80 : 64 or 5 : 4.

43.
$$160000\left(\frac{1+r}{100}\right)^3 = 185220;$$
$$\therefore \left(\frac{1+r}{100}\right)^3 = \tfrac{185220}{160000};$$
$$\therefore \frac{1+r}{100} = \tfrac{21}{20} = \tfrac{105}{100};$$
$$\therefore r = 5.$$

Hence, increase = 5%.

Population 3 yr. hence = 185220 × $(1.05)^3$
= 214415.3025;

∴ hence, population = 214415.

44. Sum remaining after spending $7\frac{1}{2}$% of money
$$= \frac{92\frac{1}{2}}{100} \text{ of money.}$$

Sum remaining after spending $4\frac{3}{4}$% of remainder
$$= \frac{95\frac{1}{4}}{100} \text{ of } \frac{92\frac{1}{2}}{100} \text{ of money} = \$1200;$$
$$\therefore \frac{95\frac{1}{4}}{100} \text{ of } \frac{92\frac{1}{2}}{100} \text{ of money} = \$1200,$$
$$\text{and money} = \frac{100}{95\frac{1}{4}} \text{ of } \frac{100}{92\frac{1}{2}} \text{ of } \$1200 = \$1361.99.$$

45. Since 75% of farm is arable, 25% of farm is the remainder. Since 83% of 25% of farm is pasture, 17% of 25% of farm is swamp;

$\therefore \frac{17}{100}$ of $\frac{25}{100}$ of farm $= 75$ ha.;
$\therefore$ entire " $= \frac{100}{17}$ of $\frac{100}{25}$ of 75 ha.
$= 1764\frac{12}{17}$ ha.

46. Since $\frac{1}{10}$ of first $= \frac{3}{20}$ of second $= \frac{1}{6}$ of third;
$\therefore$ " $= \frac{3}{2}$ " " and third $= \frac{9}{10}$ of second.
Hence, $\frac{3}{2}$ of second + second + $\frac{9}{10}$ of second $= 340$;
$\therefore \frac{34}{10}$ " " $= 340$,
and " $= 100$.
The first $= \frac{3}{2}$ of $100 = 150$.
The third $= \frac{9}{10}$ of $100 = 90$.

EXERCISE XXV. PAGE 124

6. Selling price $= \frac{9}{8}$ of 76c.
" " $= \frac{19}{20}$ of $\frac{9}{10}$ of $\frac{4}{5}$ list price;
$\therefore \frac{19}{20}$ of $\frac{9}{10}$ of $\frac{4}{5}$ list price $= \frac{9}{8}$ of 76c.;
$\therefore$ " " $= \frac{20}{19}$ of $\frac{10}{9}$ of $\frac{5}{4}$ of $\frac{9}{8}$ of 76c.
$= \$1.25$.

12. Cost to teacher $= \frac{9}{10}$ of $\frac{19}{20}$ marked price;
$\therefore \frac{9}{10}$ of $\frac{19}{20}$ marked price $= \$6.84$;
$\therefore$ " " $= \frac{10}{9}$ of $\frac{20}{19}$ of $\$6.84 = \$8$.

16. Selling price of goods $= \frac{6}{5}$ of $\$202.50 = \$243$.
Hence, $\frac{9}{10}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of list price $= \$243$;
$\therefore$ " " $= \frac{10}{9}$ of $\frac{5}{4}$ of $\frac{4}{3}$ of $\$243 = \$450$.

20. Net price of a shovel $= \frac{7}{8}$ of $\frac{4}{5}$ of $\$1.25. = 87\frac{1}{2}$c.;
$\therefore$ discount $= \$(1.25 - .87\frac{1}{2}) = 37\frac{1}{2}$c.
Hence, number of shovels bought $= \$\dfrac{12.50}{.37\frac{1}{2}} = 300$.

22. Gas consumed $= (92 \times 3\frac{1}{2} \times 12 \times 4.5)$ cu. ft.

$$\text{Net price} = \$\frac{1.25}{1000} \times 92 \times 3\tfrac{1}{2} \times 12 \times 4.5 \times \tfrac{60}{100} = \$13.041.$$

24. Marked price $= \frac{4}{3}$ of cost price.
Selling " $= \frac{9}{10}$ of $\frac{4}{3}$ of cost price $= \frac{6}{5}$ of cost price;
∴ gain $= \frac{1}{5}$ cost price $= \$5.70$;
∴ cost price $= 5 \times \$5.70 = \$28.50$.

25. By adding the water 8 pt. become 9 pt.;
∴ he may give 1 pt. in 9 as discount, or $11\frac{1}{9}\%$.

26.
$$\text{Marked price} = \left(1 + \frac{\text{Rate}}{100}\right).$$
$$\text{Discount} = \frac{\text{Rate}}{100}\left(1 + \frac{\text{Rate}}{100}\right)$$
$$= \frac{\text{Rate}}{100} + \left(\frac{\text{Rate}}{100}\right)^2;$$
$$\therefore \text{loss} = \left(\frac{\text{Rate}}{100}\right)^2 = \frac{1}{16}.$$
$$\frac{\text{Rate}}{100} = \frac{1}{4}.$$
$$\text{Rate} = 25\%.$$

27. If x represents the rate of discount,
$$\text{then } \frac{100 - x}{100} \text{ of } \frac{100 - x}{100} = \frac{4}{5} \text{ of } \frac{3}{4},$$
$$\text{and } \frac{100 - x}{100} = \sqrt{.6}$$
$$= .77459.$$
$$\therefore x = 100 - 77.459 \ldots$$
$$= 22.54 \ldots$$

Exercise XXVI. Page 128

6.
$$\text{Cost of } 54.87 \text{ cwt.} = 54.87 \times \$96$$
$$= \$5267.52;$$
$$\therefore \text{gain on } \$5267.52 = \$732.48;$$
$$\therefore \text{" " } \$100 = \$\frac{100 \times 732.48}{5267.52}$$
$$= \$13.9 \ldots;$$
$$\therefore \text{gain} = 13.9 \ldots \%.$$

22. Cost of 1 lb. of mixture $= \frac{3}{4}$ of \$1.56$\frac{2}{3}$ = \$1.17$\frac{1}{2}$.
4 lb. at \$1.30 cost 50c. more than average.
1 " \$1.05 costs .12$\frac{1}{2}$c. less " "

$$\therefore \text{ number of pounds at \$1.05} = \frac{50}{12\frac{1}{2}} = 4.$$

25. Cost of 3 lb. at 61$\frac{2}{3}$c. = \$1.85.
" " 1 " 55c. = \$0.55;

$$\therefore \text{ cost of 1 lb. of mixture} = \$\frac{2.40}{4} = 60\text{c.}$$

Gain on an outlay of 60c. = 20c.;

$$\therefore \text{ " " " } 100\text{c.} = \frac{100 \times 20}{60}\text{c} = 33\tfrac{1}{3}\text{c.};$$

$\therefore$ gain $= 33\frac{1}{3}\%$

26. Number of bushels bought = 337$\frac{1}{2}$;
$\therefore$ cost of apples = 150 × \$1.50 + \$14.40 = \$239.40.
Number of bushels sold = $\frac{4}{5}$ of 337$\frac{1}{2}$ = 270;

$$\therefore \text{ selling price of 1 bu.} = \frac{\frac{9}{8} \text{ of \$239.40}}{270} = 99\tfrac{3}{4}\text{c.}$$

28. Lot cost *B* $\frac{100}{115}$ of \$322.
" " *A* $\frac{100}{125}$ " $\frac{100}{115}$ of \$322 = \$224.

29. *C*'s buying price $= \frac{125}{100}$ of $\frac{120}{100}$ of *A*'s buying price
$= \frac{3}{2}$ of *A*'s buying price;
$\therefore$ $\frac{1}{2}$ *A*'s " " = \$180;
$\therefore$ *A*'s " " = \$360.

30. 14 oz. sell for 140% of cost.
16 " " " 160% " "
$\therefore$ his total gain = 60% " "
$\therefore$ $\frac{3}{5}$ of cost = \$240;
$\therefore$ " = \$400.

31. Selling price $= \frac{4}{5}$ of cost.
Sum received from $\frac{1}{4}$ of goods $= \frac{1}{4}$ of $\frac{4}{5}$ cost $= \frac{1}{5}$ cost.
Sum to be realized when all the goods are sold is $\frac{6}{5}$ cost;
$\therefore$ the remaining goods must sell for $(\frac{6}{5} - \frac{1}{5})$ cost.
At the old selling price the remaining goods would produce $\frac{3}{5}$ cost.
Hence, $\frac{3}{5}$ cost must be increased to $\frac{5}{5}$ cost, or by 66$\frac{2}{3}$%.

32. Cost of first lot $=\frac{3}{4}$ of selling price.
" second " $=\frac{3}{2}$ " "
$\therefore$ " both $=2\frac{1}{4}$ " "
$\therefore$ loss $=\frac{1}{4}$ of the selling $=\$60$;
$\therefore$ selling price $=\$240$.
Hence, cost of first lot $=\frac{3}{4}$ of $\$240=\$180$,
and cost of second " $=\frac{3}{2}$ " $\$240=\$360$.

33. Selling price $=\frac{130}{100}$ of cost.
But supposed cost $=\frac{9}{10}$ "
$\therefore$ $\frac{130}{100}$ of $\frac{9}{10}$ of " $=\frac{120}{100}$ " $-\$24$;
$\therefore$ $\frac{117}{100}$ " " $=\frac{120}{100}$ " $-\$24$;
$\therefore$ $\frac{3}{100}$ " " $=\$24$;
$\therefore$ " $=\$800$.

34. Since $\frac{2}{3}$ of gain on $12=$ selling price of 4;
$\therefore$ " " $12=$ " " " 6.
Selling price of $6=$ cost price of 12.
" " " $1=$ " " " 2.
Gain $=100\%$.

36. Cost of 1 apple of first kind $=\frac{1}{3}d.$
" 1 " " second " $=\frac{1}{2}d.$;
$\therefore$ average cost of 1 apple $=\dfrac{\frac{1}{3}+\frac{1}{2}}{2}d.=\frac{5}{12}d.$
Selling price " 1 " $=\frac{2}{5}d.$;
$\therefore$ loss on an outlay of $\frac{5}{12}d.=(\frac{5}{12}-\frac{2}{5})d.=\frac{1}{60}d.$;
$\therefore$ " " $100d.=\dfrac{100\times\frac{1}{60}}{\frac{5}{12}}d.=4d.$;
$\therefore$ she lost 4%.

41. On $100 outlay he should get $160, but receives only three-eighths of $160, that is, $60. Thus he loses 40%.

42. His gross outlay is decreased 35% by the fall in flour, and 5% by the lowering of trade expenses, and, therefore, 40% in all.

Hence, he can lower the 15c. loaf 40%, that is by $\frac{2}{5}$ of 15c., or 6c.

44. $\$38.25 = 92\%$ of cost.

$\$57 = \frac{5700}{3825} \times \frac{92}{1}\%$ of cost $= 137\frac{5}{51}\% \ldots$;

$\therefore$ gain $= 37\frac{5}{51}\%$.

45. Asking price $= \frac{125}{100}$ of cost price;

$\therefore$ selling " $= \frac{85}{100}$ of $\frac{125}{100}$ of cost price

$= \frac{106\frac{1}{4}}{100}$ of cost price;

$\therefore$ $\frac{6\frac{1}{4}}{100}$ of cost price $= \$5.75$;

$\therefore$ " " $= \$\frac{100 \times 5.75}{6\frac{1}{4}} = \$92$,

and asking price $= \frac{125}{100}$ of $\$92 = \$115$.

46. Cost price $= 20 \times 16 \times 55$c. $= \$176$.

Number of Troy ounces bought $= \frac{20 \times 7000}{20 \times 24}$.

Selling price $= \frac{20 \times 7000 \times 60}{20 \times 24}$c. $= \$175$;

$\therefore$ loss $= \$1$.

49. Amount of \$6.30 at end of 6 mo.

$= \$\frac{6.30 \times 104}{100} = \$6.552$;

$\therefore$ buying at \$6.50 on 6 mo. is the more profitable.

Or present value of $\$6.50 = \$\frac{6.50 \times 100}{104} = \$6.25$;

$\therefore$ buying at \$6.50 on 6 mo. is the more profitable.

54. Cost of first horse $= \$\frac{100 \times 100}{125} = \$80$.

" second " $= \$\frac{100 \times 100}{75} = \$133\frac{1}{3}$;

$\therefore$ he loses $\$(80 + 133\frac{1}{3} - 200) = \$13\frac{1}{3}$.

56. Cost price of 1 lb. $= \frac{72 \times 100}{108} = 66\frac{2}{3}$c.;

$\therefore$ " " 225 " $= 225 \times 66\frac{2}{3}$c. $= \$150$.

Sum to be realized $= \$\frac{150 \times 110}{100} = \$165$.

Selling price of 45 lb. $= 45 \times 72$c. $= \$32.40$;
$\therefore$ " " $(225-45)$ " $= \$132.60$;
$\therefore$ " " 1 " $= \$\dfrac{132.60}{180} = 73\tfrac{2}{3}$c.

58. The difference in price, $\$\tfrac{2}{3}$, must be divided in proportion to $2\tfrac{1}{3}$ and $1\tfrac{2}{3}$, or as 7 to 5.

$$\tfrac{7}{12} \text{ of } \$\tfrac{2}{3} = \$\tfrac{7}{18} \text{ and } \tfrac{5}{12} \text{ of } \$\tfrac{2}{3} = \$\tfrac{5}{18}.$$

Hence, the selling price is $\$2\tfrac{1}{3} - \$\tfrac{7}{18}$, or $\$1\tfrac{2}{3} + \$\tfrac{5}{18}$, or $\$1\tfrac{17}{18}$.
Again, a yard of each kind cost \$4 and sold for $\$3\tfrac{8}{9}$, or $2 \times \$1\tfrac{17}{18}$;
$\therefore$ loss on \$4 $= \$\tfrac{1}{9}$;
$\therefore$ " " $\$100 = \$\dfrac{100 \times \frac{1}{9}}{4} = \$2\tfrac{7}{9}$;
$\therefore$ " per cent. $= 2\tfrac{7}{9}$.

62. Selling price $= 75\%$ of cost;
$\therefore$ $102\tfrac{7}{9}\%$ of cost $= 75\%$ " $+ \$500$.
$27\tfrac{7}{9}\%$ " $= \$500$.
100% " $= \$1800$.

63. $7\tfrac{1}{2}\%$ of cost of first $= 5\%$ of \$(4000 − cost of first)
$= \$200 - 5\%$ of " "
$\therefore$ $12\tfrac{1}{2}\%$ " " " $= \$200$.
" " $= \$1600$.

65. Sum realized from first sale $= \tfrac{75}{100}$ of sum invested.
" " " second " $= \tfrac{125}{100}$ of $\tfrac{75}{100}$ " "
$= \tfrac{15}{16}$ " "
$\therefore$ loss $= \tfrac{1}{16}$ " "
or $6\tfrac{1}{4}\%$.

66. Sum realized from first sale $= \tfrac{112}{100}$ of investment.
" " " second " $= \tfrac{115}{100}$ of $\tfrac{112}{100}$ " "
" " " third " $= \tfrac{75}{100}$ " $\tfrac{115}{100}$ " $\tfrac{112}{100}$ "
$= \tfrac{483}{500}$ " "
$\therefore$ $\tfrac{483}{500}$ of investment $= \$1254.80$;
$\therefore$ " $= \$\dfrac{500 \times 1254.80}{483}$
$= \$1298.96$, nearly.

67. Selling price of manufacturer $= \frac{120}{100}$ of cost.
" " " wholesaler $= \frac{125}{100}$ of $\frac{120}{100}$ " "
" " " retailer $= \frac{140}{100}$ " $\frac{125}{100}$ of $\frac{120}{100}$ " "
$= \frac{21}{10}$ " cost;
$\therefore$ $\frac{21}{10}$ of cost $= \$2.10$;
$\therefore$ " $= \$\frac{10 \times 2.10}{21} = \$1.$

Exercise XXVII. Page 138

5. Commission on $\$578 = \$26.01$;
$\therefore$ " " $\$100 = \$\frac{100 \times 26.01}{578} = \$4\frac{1}{2}$;
$\therefore$ rate of commission $= 4\frac{1}{2}\%$.

13. Commission on $\$100$ invested $= \$2\frac{1}{3}$;
$\therefore$ " " $\$102\frac{1}{3}$ sent $= \$2\frac{1}{3}$;
$\therefore$ " " $\$3377$ " $= \$\frac{3377 \times 2\frac{1}{3}}{102\frac{1}{3}} = \$77.$

16. Sum invested $= \$\frac{3681 \times 100}{10\text{[illegible]}}$
$= \$3600.$

Number of barrels bought $= \frac{3600}{7.50} = 480.$
Total selling price $= \frac{120}{100}$ of $\$(3681 + 119) = \$4560$
$\therefore$ selling price of 1 bbl. $= \$\frac{4560}{480} = \$9.50.$

18. If he sell wheat to the value of $100 his commission = $2 and he has $98 to invest in silk.

Commission on $\$98 = \$\frac{98 \times 4}{104} = \$3\frac{10}{13}$;
$\therefore$ total commission $= \$5\frac{10}{13} = \$\frac{75}{13} = \$\frac{600}{104}$;
$\therefore$ sum invested when $\$5\frac{10}{13}$ is the commission $= \$94\frac{3}{13}$
$\therefore$ " " " $\$600$ " "
$= \$\frac{600 \times 94\frac{3}{13}}{5\frac{10}{13}} = \$9800.$

General Rule :

Total commission $= \$\frac{600}{104} = \frac{6}{104}$ of $\$\frac{100}{1}$ when sales $= \$100$.

$$\frac{6}{104} = \frac{2+4}{100+4} = \frac{\text{commission for selling} + \text{commission for buying}}{100 + \text{commission for buying}}.$$

$\frac{6}{104}$ of sales $= \$600$.

$\frac{98}{104}$ " " $= \$9800$.

19. The first commission is charged on \$7650.

" second " " " " \$7350.

The sum on which a single commission would equal the two commissions is \$15000;

∴ commission on \$15000 = \$300;

$$\therefore \quad \text{"} \quad \text{"} \quad \$100 = \$\frac{100 \times 300}{15000} = \$2;$$

∴ rate of commission = 2%.

20. $$\text{Sum invested in tea} = \$\frac{3060 \times 100}{102} = \$3000.$$

$$\text{Number of pounds bought} = \frac{3000}{.75} = 4000.$$

Selling price $= \frac{125}{100}$ of $\$(3060 + 30 + \frac{1}{300}$ of $3000)$

$= \$3875$;

∴ selling price of 1 lb. $= \$\frac{3875}{4000} = 96\frac{7}{8}$c.

21. Total commission $= \frac{7}{104}$ of sales $= \$700$.

Amount invested $= \frac{97}{104}$ " $= \frac{97}{7}$ of $\$700 = \$9700$.

(See General Rule, problem number 18.)

27. Cost of carpet $= 3200 \times 96$c. $+ \frac{3}{400}$ of 3200×96c. $+ \$105.06$

$= \$3200.10$;

∴ selling price per yard $= \frac{1}{3200}$ of $\frac{9}{8}$ of \$3200.10

$= \$1.12\frac{1}{2}$c. nearly.

29. $\frac{3}{200}$ of value of land $= \$278.40$

∴ " " $= \frac{200}{3}$ of \$278.40;

$$\therefore \text{ number of acres sold} = \frac{1}{14.50} \text{ of } \frac{200}{3} \text{ of } 278.40 = 1280.$$

30. $(2\frac{1}{2}\% + 1\frac{1}{2}\%)$, or 4% of sales $= \$564$.

33. $$\text{Commission} = \frac{4\frac{1}{2}}{101\frac{1}{2}} \text{ of sales.}$$
$$\text{Amount invested} = \frac{97}{101\frac{1}{2}} \text{ of sales.}$$
$$\text{Value of coal oil} = \frac{97}{101\frac{1}{2}} \text{ of \$406.}$$

34. Sums on which commission were charged are \$6120 and \$5760.

The total commission must be the same as if the same rate were charged on \$5760 and on 2 × \$6120, or on \$18000.

Hence, commission on \$18000 = \$360,
and " " $\$100 = \frac{100}{18000}$ of \$360 = \$2.
Hence, the rates are 4% and 2%.

35. Total commission = \$100;
∴ amount of sales exceeds amount invested by \$100;
∴ commission for selling exceeds commission for buying by the commission on \$100;
∴ commission on \$100 = \$4. Rate is 4%.

36. $\text{Commission} = \frac{1}{8} \text{ of } (27 \times \$175 + 25 \times \$85 + 40 \times \$45 + 50 \times \$20)$
$= \$1206.25.$
Five dollars on each implement = 142 × \$5 = \$710.
Income = \$1206.25 + \$710 = \$1916.25.

37. Sum remaining after first discount $= \frac{70}{100}$ of list price.
" " " second " $= \frac{95}{100}$ of $\frac{70}{100}$ of list price.
Sum necessary to pay the bill and the agent's commission
$= \frac{105}{100}$ of $\frac{95}{100}$ of $\frac{70}{100}$ of list price;
∴ $\frac{105}{100}$ of $\frac{95}{100}$ of $\frac{70}{100}$ of list price = \$837.90,
and list price $= \frac{100}{105}$ of $\frac{100}{95}$ of $\frac{100}{70}$ of \$837.90
= \$1200.
Sum charged for commission $= \frac{5}{100}$ of $\frac{95}{100}$ of $\frac{70}{100}$ of \$1200
= \$39.90.
Or, $\frac{105}{100}$ of net price of goods = \$837.90,
and the commission $= \frac{5}{105}$ of \$837.90
= \$39.90.

38. Value of butter sold $= (60 \times 56 \times 20)$c. $= \$672$.

Sum remaining after deducting first commission and shipping expenses $= \$(672 - \frac{4}{100}$ of $672 - 36.12)$

$= \$609$.

Sum remaining after deducting second commission

$= \dfrac{100}{101\frac{1}{2}}$ of $\$609 = \$600$.

Number of pounds of rice bought $= \frac{60000}{3}$ lb. $= 20000$ lb.

Total commission $= \$26.88 + \$9$

$= \$35.88$.

Exercise XXVIII. Page 145

3. Premium on £100 at $2\frac{1}{3}\% = £2\frac{1}{3}$;

$\therefore$ sum for which goods worth $£97\frac{2}{3}$ are insured $= £100$;

$\therefore$ " " " " " $£4384\frac{1}{80}$ " "

$= £\dfrac{4384\frac{1}{80} \times 100}{97\frac{2}{3}}$

$= £4488$ 15*s*.

5. Total payment for cargo worth \$100 $= \$(1\frac{1}{2} + \frac{1}{8} + \frac{1}{6})$

$= \$1\frac{19}{24}$;

$\therefore$ " " " " " $\$26400 = \$(264 \times 1\frac{19}{24})$

$= \$473$.

6. \$2.20 is premium on \$100;

$\therefore$ \$80.85 " " " $\$\dfrac{80.85 \times 100}{2.20} = \$3675$.

But \$3675 is $\frac{75}{100}$ of value of 500 bbl. of flour;

$\therefore$ value of 1 bbl. $= \$\dfrac{\frac{100}{75} \times 3675}{500} = \$9.80$.

7. $\frac{5}{200}$ of risk $= \frac{3}{100}$ of $\frac{4}{5}$ of risk $+ \$10$;

$\therefore \frac{5}{200}$ " " $= \frac{12}{500}$ of risk $+ \$10$;

$\therefore (\frac{5}{200} - \frac{12}{500})$ " " $= \$10$;

$\therefore \dfrac{25 - 24}{1000}$ " " $= \$10$;

$\therefore$ " $= \$(1000 \times 10) = \$10000$.

11. The broker first offered $\frac{9}{10}$ of the value;

$$\text{then } \tfrac{9}{10} \text{ of the value} + \$379.75 = \frac{105\frac{1}{2}}{100} \text{ of the value;}$$

$$\therefore\ \$379.75 = (\tfrac{211}{200} - \tfrac{9}{10}) \text{ of the value} = \tfrac{31}{200} \text{ `` `` ``}$$

$$\therefore\ \text{the value} = \tfrac{200}{31} \text{ of } \$379.75 = \$2450.$$

13. Premium paid on $\frac{2}{3}$ of \$75000 = \$(377.50 − 250);

$$\therefore\ \text{`` `` `` } \$100 = \frac{100}{\frac{2}{3} \text{ of } 75000} \text{ of } \$375. = \$\tfrac{3}{4};$$

$$\therefore\ \text{rate of insurance} = \tfrac{3}{4}\%.$$

14. Premium paid the Ottawa Co. = $\frac{3}{200}$ of $\frac{2}{3}$ of \$75000 = \$750.
Premium paid British American Co. = $\frac{5}{400}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of \$75000
= \$312.50.
Loss sustained by Ottawa Co. = $\frac{1}{2}$ of $\frac{2}{3}$ of \$75000
+ \$312.50 − \$750 = \$24562.50.
Loss of owners = $\frac{1}{3}$ of \$75000 + \$750
= \$25750.

15.

PROPERTY	VALUE	INSURANCE	DAMAGE	CLAIM
Building . . .	\$40000	\$20000	\$25000	\$20000
Machinery . .	60000	35000	30000	30000
Stock	40000	25000	20000	20000
			\$75000	\$70000

Claim = \$70000.
Owner's loss = \$(75000 − 70000) = \$5000.

17. Buildings are equal in value.

$$1\tfrac{1}{4}\% \text{ of value} = \$90.$$

$$\tfrac{3}{4}\% \text{ `` } = \$\tfrac{90}{1} \times \tfrac{4}{5} \times \tfrac{3}{4} = \$54.$$

19. Premium on \$5400 = \$(5400 − 5300 − 4.60) = \$95.40;

$$\therefore\ \text{`` `` } \$100 = \tfrac{100}{5400} \text{ of } \$95.40 = \$1\tfrac{23}{30};$$

$$\therefore\ \text{rate} = .1\tfrac{23}{30}\%.$$

EXERCISE XXIX. PAGE 150

4. Of each \$100 collected, \$96 is spent in paying for the school-house;

$\therefore$ \$8400 requires a tax of $\$\dfrac{8400 \times 100}{96} = \$8750.$

Tax paid by $\$700000 = \$8750;$

$\therefore$ " " " $\$1 = \frac{875000}{700000}\text{c.} = 1\frac{1}{4}\text{c.}$

5. Tax on $\$1100 = \$(1800 - 1781.85) = \$18.15;$

$\therefore$ " " $\$1 = \$.0165;$

$\therefore$ rate on dollar $= 16\frac{1}{2}$ mills.

7. \$.016 is the tax on \$1;

$\therefore$ \$25.60 " " $\$\dfrac{25.60 \times 1}{.016}$, or \$1600;

$\therefore$ my income is \$(1600 + 700), or \$2300.

8. Premium paid $= \frac{7}{400}$ of $\frac{2}{3}$ of value $= \$52\frac{1}{2};$

$\therefore$ value $= \$\dfrac{400 \times 3 \times 52\frac{1}{2}}{7 \times 2} = \$4500.$

Tax on $\$1 = \$.0175;$

$\therefore$ tax on $\frac{4}{5}$ of $\$4500 = \$(\frac{4}{5}$ of $4500 \times .0175) = \$63.$

11. $\frac{1}{4}$c., or $\$\frac{1}{400}$ = difference of tax on \$1;

$\therefore$ \$3.60 = " " " $\$\dfrac{3.60 \times 1}{\frac{1}{400}} = \$1440.$

13. $\frac{98}{100}$ of tax levied $= \$19600;$

$\therefore$ " " $= \frac{100}{98}$ of $\$19600 = \$20000.$

Hence, $\frac{5}{1000}$ of value of property $= \$20000;$

$\therefore$ " " " $= \frac{1000}{5}$ of $\$20000 = \$4000000.$

14. Rate of taxation $= \$\frac{10200}{850000} = 12$ mills.

Tax on 1 section $= \$(640 \times 12\frac{1}{2} \times .012) = \$96.00.$

15. $\$\frac{14}{1000}$ is the tax on \$1;

$\therefore$ \$28 " " $\$\dfrac{28 \times 1000}{14}$, or on \$2000.

Hence, Smith's capital $= 3 \times \$2000 = \$6000.$

16. Since $\frac{95}{100}$ of tax is collected, and of this $\frac{2}{100}$ is paid for collection,

$\therefore$ $\frac{98}{100}$ of $\frac{95}{100}$ of the tax = \$558600;

$\therefore$ " " $= \$\dfrac{100 \times 100 \times 558600}{98 \times 95} = \$600000.$

Rate of taxation $= \$\frac{600000}{50000000} = 12$ mills.

20. $\frac{96}{100}$ of tax levied = \$43200;

$\therefore$ " " $= \frac{100}{96}$ of \$43200 = \$45000.

Hence, rate $= \$\frac{45000}{2500000} = 18$ mills.

22. $\dfrac{8\frac{3}{4}}{1000}$ of assessed value of property = \$2406.25;

$\therefore$ " " " $= \$\dfrac{1000 \times 2406.25}{8\frac{3}{4}}$

= \$275000.

Exercise XXX. Page 154

4. Cost of 8400 lb. = \$630.

Specific duty = 8400 × ½c. = \$42.

Ad valorem " $= \$\dfrac{630 \times 35}{100} = \$220.50;$

$\therefore$ total " = \$262.50.

6. Value of the cabinet organs = $\frac{100}{25}$ of \$196.50 = \$786.

My outlay = \$(786 + 196.50);

$\therefore$ selling price = $\frac{120}{100}$ of \$982.50 = \$1179.

10. Total duty = 25 × \$2.40 + $\frac{3}{10}$ of \$80 = \$84.

11. Specific duty = 40 × \$2.40 = \$96.

Ad valorem " = \$(159 − 96) = \$63;

$\therefore$ $\frac{3}{10}$ of the value of spirits = \$63;

$\therefore$ " " " $= \frac{10}{3}$ of \$63 = \$210.

12. Duty charged = \${1198 − (840 + 64)} = \$294;

$\therefore$ duty on \$840 = \$294;

$\therefore$ " " \$100 = $\frac{100}{840}$ of \$294 = \$35.

Hence, the rate of duty is 35%.

16. Cost of piano and the duty $= \$(372 - 7.50)$
$= \$364.50$;
∴ $\frac{135}{100}$ of invoice price of piano $= \$364.50$;
∴ " " " $= \frac{100}{135}$ of $\$364.50 = \$270$.

17. Cost of a watch case $= \$15 + \frac{3}{10}$ of $\$15 = \$19.50$.
Selling price at 20% profit $= \frac{6}{5}$ of $\$15 = \$18$;
∴ reduction when there is no duty $= \$1.50$.

18. Specific duty $= 150 \times \$2.40 = \$360$.
Ad valorem " $= \$(414 - 360) = \$54$.
Duty on $150 \times \$1.20 = \$54$;
∴ " $100 $= \$\frac{100 \times 54}{150 \times 1.20} = \$30$;
∴ the rate of duty is 30%.

19. Ad valorem duty $= \frac{1}{4}$ of $\frac{30000}{1000} \times \$60 = \$450$.
Specific " $= \$(1425 - 450) = \$975$.
Specific duty on 325 lb. $= \$975$;
∴ " 1 lb. $= \$\frac{975}{325} = \$3$.

EXERCISE XXXI. PAGE 159

1.

DATE	RECEIVED	DELIVERED	BALANCE	DAYS	PRODUCTS
Sept. 16 . .	400		400	4	1600
" 20 . .	150		550	13	7150
Oct. 3 . .	200		750	9	6750
" 12 . .	240		990	8	7920
" 20 . .	300		1290	13	16770
Balance on hand Nov. 2.	1290				30)40190 1339⅔

The storage of the various quantities of barrels is the same as the storage of 1339⅔ bbl. for 30 da.;

∴ storage bill $= 1339\frac{2}{3} \times 4$c. $= \$53.58\frac{2}{3}$.

2.

DATE	RECEIVED	DELIVERED	BALANCE	DAYS	PRODUCTS
Jan. 5 . .	150		150	8	1200
“ 13 . .		100	50	12	600
“ 25 . .	200		250	5	1250
“ 30 . .		150	100	11	1100
Feb. 10 . .	250		350	5	1750
“ 15 . .	300		650	5	3250
“ 20 . .		200	450	9	4050
March 1 . .		450			
	900	900			30)13200 440

The storage of 440 bbl. for 1 mo. at 5c. per mo. is $22.

3.

DATE	RECEIVED	DELIVERED	BALANCE FOR SECOND MONTH	STORAGE DUES
June 10 . . .	250			$12.50
“ 25 . . .		150	100	3.00
July 15 . . .	200			10.00
“ 20 . . .		80		
“ 25 . . .		150		
“ 28 . . .	150			7.50
Aug. 3 . . .	250			12.50
“ 5 . . .		400		
“ 20 . . .		70		
	850	850		$45.50

4.

DATE	RECEIVED	DELIVERED	BALANCE FOR SECOND MONTH	STORAGE DUES
May 10 . . .	250			$12.50
" 25 . . .		150		
" 24 . . .	200			10.00
June 2 . . .		100		
" 12 . . .	200			10.00
" 18 . . .		300		
" 18 . . .	150			7.50
" 30 . . .		250		
	800	800		$40.00

EXERCISE XXXII. PAGE 161

7. Interest on $\$100$ for 8 mo. at $5\% = \$3\frac{1}{3}$;

∴ principal which amounts to $\$1 = \$\frac{100}{103\frac{1}{3}}$;

∴ " " " " $\$1395 = \$\frac{1395 \times 100}{103\frac{1}{3}}$

$= \$1350.$

8. Sum on which $\$54.00$ is interest $= \$100$;

∴ " " $\$202.50$ " " $= \$\frac{202.50 \times 100}{54}$

$= \$375.$

11. $\$170$ is amount of $\$100$ in 10 yr. at 7%;

∴ $\$1275$ " " $\frac{100}{170} \times \frac{1275}{1}$ in 10 yr. at $7\% = \$750$.

$\$52.50$ is interest on $\$750$ for 1 yr. at 7%;

∴ $\$(1406.25 - 750)$ is interest on $\$750$ for $\frac{656.25 \times 1}{52.50}$ yr.

$= 12\frac{1}{2}$ yr.

12. The interest on \$400 for 3 mo. = the interest on \$100 for 12 mo.

The interest on \$100 for 12 mo. at a certain rate per cent. = the interest on \$200 for 12 mo. at half that rate;

∴ the sum borrowed would pay the same interest as \$(500 + 200) would.

Interest on \$700 for 1 yr. = \$35;

∴ " \$100 " 1 " $= \frac{100 \times 35}{700} =$ \$5;

∴ rate = 5%.

17. \$94.7625 is interest for 365 da.;

∴ \$37.905 " " " $\frac{37.905 \times 365}{94.7265}$ da. = 146 da.

19. The interest for 2½ yr. = \$(347.50 − 310) = \$37.50;

∴ " " " 4 " $= \$\frac{4 \times 37.50}{2\frac{1}{2}}$ = \$60;

∴ the principal = \$(310 − 60) = \$250.

Since the interest on \$250 for 4 yr. = \$60,

∴ " " " \$100 " 1 " $= \$\frac{100 \times 60}{250 \times 4} =$ \$6;

∴ rate = 6%.

20. The interest at 1¼% for the time = \$(416.25 − 405)
= \$11.25;

∴ " " " 5% " " " $= \$\frac{5 \times 11.25}{1\frac{1}{4}}$ = \$45;

hence, the principal = \$(405 − 45) = \$360.

\$18 is interest for 1 yr. on \$360 at 5%.
\$45 " " $\frac{45}{18}$ " " " " "
Time 2½ " " " " "

28. The interest on \$2000 for 3 mo. = \$37.50;

∴ at the end of 2 yr. the second would have received \$19000 + 7 × \$37.50 + 6 × \$37.50 + 5 × \$37.50 + 4 × \$37.50 + 3 × \$37.50 + 2 × \$37.50 + \$37.50, or \$20050, which is more than the first tender by \$50.

32. Number of years $= 100 \div 3\frac{1}{3} = 30$.

33. Number of days between May 1st and Nov. 14th $= 197$.
Interest on \$1200 for 197 da. at 8% $= \$(\frac{197}{365}$ of $\frac{8}{100}$ of $1200)$
$= \$51.81$.
Sum that will pay the debt $= \$1200 + \$51.81$
$= \$1251.81$.

34. Interest on \$750 for $1\frac{1}{2}$ yr. $= \$67\frac{1}{2}$;
∴ " " \$1250 " 2 " $= \frac{1250}{750}$ of $\dfrac{2}{1\frac{1}{2}}$ of $\$67\frac{1}{2} = \$150$.

36. $\frac{13}{200}$ of sum invested $+ \frac{15}{1000}$ of $\frac{2}{3}$ of sum invested $= \$360$;

$$\therefore \text{ sum invested} = \frac{\$360}{\frac{13}{200} + \frac{15}{1000} \text{ of } \frac{2}{3}} = \$4800.$$

37. The interest on \$800 for 1 yr. + the interest on \$1200 for a year at double the rate = the interest on \$800 + the interest on \$2400 at the same rate;
∴ the interest on \$3200 for 1 yr. $= \$136$;
∴ " " " \$100 " 1 " $= \frac{1}{32}$ of $\$136 = \$4\frac{1}{4}$;
∴ the rates are $4\frac{1}{4}$% and $8\frac{1}{2}$%.

38. Interest on \$2400 for $1\frac{3}{5}$ yr. at 8% $= 1\frac{3}{5} \times \frac{8}{100}$ of \$2400
$= \$307.20$.
Now \$25 is the interest on \$100 for $3\frac{1}{3}$ yr. at $7\frac{1}{2}$%;
∴ \$307.20 is the interest on $\dfrac{307.20}{25}$ of \$100, or on \$1228.80.

39. Tax on property $= \dfrac{17\frac{1}{2}}{1000}$ of $\$45000 = \$787.50$.
\$6 is the interest on \$100.
\$787.50 " " " $\$\dfrac{787.50 \times 100}{6} = \$13125$.

40. Sum realized $= \dfrac{106\frac{1}{4}}{100}$ of $\$5000 = \$5312.50$.
Now, the interest for 3 mo. at 3% $= \frac{3}{400}$ of principal,
and the amount " 3 " 3% $= \frac{403}{400}$ " "
∴ $\frac{403}{400}$ of principal deposited $= \$5000$;
∴ " " $= \frac{400}{403}$ of $\$5000 = \$4962.77$;
∴ sum remaining $= \$(5312.50 - 4962.77) = \$349.73$.

EXERCISE XXXIII. PAGE 171

1. (*b*) The face value of the note on June 24, is $600 + interest on $600 for 123 da., at 6%, or $612.13.

The term of discount is 73 da.

The bank discount for 73 da. on $612.13 at 8%

$$= \tfrac{73}{365} \text{ of } \tfrac{8}{100} \text{ of } \$612.13 = \$9.79\ldots$$

The proceeds $= \$(612.13 - 9.79) = \$602.34$.

3. Since the interest on $100 for 3 mo. at 8% = $2,

∴ note of which the proceeds are $98 = $100;

" " " " " " $700

$$= \$\frac{700 \times 100}{98} = \$714.28.$$

4. Since the interest on $100 for 3 mo. at 10% = $2\tfrac{1}{2}$,

∴ note of which the proceeds are $\$97\tfrac{1}{2} = \$100$;

" " " " " " $1300

$$= \$\frac{1300 \times 100}{97\frac{1}{2}} = \$1333\tfrac{1}{3}.$$

6. Sum loaned + $\tfrac{3}{4}$ of $\tfrac{1}{10}$ of sum loaned = face of note;

∴ " " $= \tfrac{40}{43}$ of face of note;

∴ sum exacted for 9 mo. $= \tfrac{3}{43}$ " "

∴ " " " 12 " $= \tfrac{4}{43}$ " "

∴ rate $= 9\tfrac{13}{43}\%$.

7. The bill was paid by $(864 – 42.20), or $8

The bank discount on the note $= \tfrac{125}{365}$ of $\tfrac{6}{100}$ of note

$= \tfrac{3}{146}$ of note;

∴ the proceeds $= \tfrac{143}{146}$ "

∴ $\tfrac{143}{146}$ of note = $820.80;

$$\therefore \text{ note} = \$\frac{146 \times 820.80}{143} = \$838.02\ldots;$$

∴ gain = $(864 – 838.02) = $25.98.

13. The interest on $1000 at 5% for 1 yr. = $50.

$50 is the discount for 1 yr.;

∴ $10 " " " $\tfrac{10}{50}$ yr., or 73 da.

14. Interest on \$100 for 60 da. at $5\frac{1}{4}\% = \$\frac{63}{73}$;
∴ proceeds are $\$99\frac{10}{73}$ when face of note is \$100;
∴ “ “ \$289.48 “ “ “ “ $\$\frac{289.48 \times 100}{99\frac{10}{73}}$,
or \$292.

19. Discount $= \frac{50}{365}$ of $\frac{73}{800}$ of face value $= \frac{1}{80}$ of face value;
∴ $\frac{79}{80}$ “ “ $= \$308.10$,
and face value $= \frac{80}{79}$ of $\$308.10 = \$312$.

20. Interest on \$2190 for 60 da. $= \$(2190 - 2163)$;
∴ “ “ \$100 “ 365 “ $= \frac{100}{2190}$ of $\frac{365}{60}$ of $\$27 = \$7\frac{1}{2}$;
∴ rate of discount is $7\frac{1}{2}\%$.

24. The discount $= \frac{1}{5}$ of $\frac{1}{10}$ of face value
$= \frac{1}{50}$ of face value;
∴ net proceeds $= \frac{49}{50}$ “ “

Hence when the net proceeds are \$98 for 73 da. the interest is \$2;
∴ when the net proceeds are \$100 for 365 da. the interest is $\frac{100}{98}$ of $\frac{365}{73}$ of \$2, or $\$10\frac{10}{49}$.

Hence the rate charged is $10\frac{10}{49}\%$.

EXERCISE XXXIV. PAGE 177

4. The simple interest for 3 yr. at $5\% = \frac{15}{100}$ of principal;
∴ $\frac{15}{100}$ of principal $= \$126$;
∴ principal $= \frac{100}{15}$ of $\$126 = \$840$.
Amount of \$1 $= \$(1.05)^3 = \$1.157625$;
∴ “ “ $\$840 = 840 \times \$1.157625 = \$972.405$;
∴ compound interest $= \$972.405 - \$840 = \$132.405$.

5. The amount of $\$1 = \frac{115}{100}$ of \$1;
∴ $\frac{115}{100}$ of principal $= \$1150$;
∴ “ $= \frac{100}{115}$ of $\$1150 = \$1000$.
The interest of \$1 for 3 yr. $= \$(1.06^3 - 1) = \$.191016$.
“ “ “ \$1 “ 2 “ $= \$(1.06^2 - 1) = \$.1236$;
∴ “ “ “ \$1 “ $2\frac{1}{2}$ “ $= \$\left(.1236 + \frac{.191016 - 1236}{2}\right)$
$= \$.157308$;
∴ amount of \$1000 “ $2\frac{1}{2}$ “ $= 1000 \times \$1.157308 = \$1157.308$.

11. Amount of \$1 for 3 yr. $= \$(1.04)^3 = \$1.124864$.
" " \$1 " 2 " $= \$(1.04)^2 = \$1.0816$.
Interest during 3rd yr. $= \$.043264$;

∴ amount of \$1 for $2\frac{1}{2}$ yr. $= \$\left(1.0816 + \frac{.043264}{2}\right)$
$= \$1.103232$.

Hence, \$1.103232 is amount of \$1 in $2\frac{1}{2}$ yr. at 4%;
∴ \$16989.7728 " " " $\$\frac{16989.7728}{1.103232}$,
or \$15400.

13. The compound interest on \$1 $= \$(1.05^2 - 1) = \$.1025$.
" simple " " \$1 $= \$.10$.
\$.0025 is difference when principal $= \$1$.
\$3 " " " " $= \$\left(1 \times \frac{3}{.0025}\right)$
$= \$1200$.

14. The interest on \$36.40 for 1 yr. $= \$(37.856 - 36.40)$
$= \$1.456$;
∴ " " " \$100 " 1 " $= \$\frac{100 \times 1.456}{36.40} = \$4$;
∴ rate = 4%.

16. Since the interest for the fourth year is \$2.205 greater than for the third year, and for the third it is \$2.10 greater than for the second year;
∴ the interest on \$2.10 for 1 yr. $= \$.105$;
∴ " " " \$100 " 1 " $= \$\frac{100 \times .105}{2.10}$
$= \$5$;
∴ rate = 5%.
Again, sum on which \$5 is the interest for 1 yr. $= \$100$;
∴ " " " \$40 " " " 1 " $= \$800$.

17. $(1.035)^2 = 1.071225$;
∴ rate = 7.1225%.

18. If rate $= r$

$$\text{Then } \left(1+\frac{r}{100}\right)^2 = 1.10;$$

$$\therefore \left(1+\frac{r}{100}\right) = \sqrt{1.10} = 1.0488\ldots;$$

$$\therefore \frac{r}{100} = .0488;$$

$$\therefore r = 4.88\%.$$

19. The amount of $\$1$ for 3 yr. at $5\% = \$(1.05)^3$;

$\therefore$ the present worth of $\$(1.05)^3 = \$1$;

$$\therefore \text{“ “ “ “ } \$648.27 = \$\frac{648.27}{(1.05)^3}$$

$$= \$560.$$

21. Interest for first year $= \$250.$

“ “ second “ $= \$275.$

“ “ third “ $= \$302.50.$

“ “ fourth “ $= \$332.75.$

“ “ fifth “ $= \$366.025.$

The sum of these is $\$1526.275$; hence, the interest to be gained after the fifth year is $\$201.31\frac{3}{8}$. But the interest for the sixth year is $\$402.62\frac{3}{4}$;

$\therefore 5\frac{1}{2}$ yr. is the time required.

22. The compound interest on $\$1 = \$(1.06^2 - 1) = \$.1236.$

“ simple “ “ $\$1 = \$.12.$

Principal on which the difference is $\$.0036 = \$1$;

$$\therefore \text{“ “ “ “ “ “ } \$9 = \$\frac{9}{.0036}$$

$$= \$2500.$$

27. The amount of $\$1 = (1.02)^4$

$$= \$1.08243\ldots;$$

$\therefore$ interest of $\$100 = 100 \times \$.08243\ldots$

$$= \$8.243\ldots;$$

$\therefore$ rate per cent. $= 8.243\ldots$

30. If r is the rate per cent., then $\$\left\{\left(1+\frac{r}{100}\right)^3-1\right\}$ is the interest on \$1 for 3 yr., and $6000\times\$\left\{\left(1+\frac{r}{100}\right)^3-1\right\}$ is the fourth year's interest on \$6000;

$$\therefore\ 6000\times\$\left\{\left(1+\frac{r}{100}\right)^3-1\right\}=\$749.184;$$

$$\therefore\ \left(1+\frac{r}{100}\right)^3-1=\frac{749.184}{6000},$$

$$\text{and } 1\times\frac{r}{100}=\sqrt[3]{1+\frac{749.184}{6000}}=1.04;$$

$$\therefore\ \frac{r}{100}=.04;$$

$$\therefore\ r=100\times .04=4.$$

Also $\frac{4}{100}$ of sum $=\$6000$;

" $=\$\frac{100\times 6000}{4}=\$150000$.

32. Sum on deposit June 1st, 1907

$=[\$400\times\{(1.05)^4+(1.05)^3+(1.05)^2+(1.05)\}]$.

34. Increase is $\{(\frac{11}{10})^3-1\}$ of former population;

$\therefore$ $\frac{331}{1000}$ of former population $=2648$;

$\therefore$ " " $=\frac{1000}{331}$ of 2648

$=8000$,

and present population $=8000+2648$

$=10648$.

37. Interest on \$54 for 1 yr. $=\$4.32$;

$\therefore$ " " \$100 " 1 " $=\frac{100}{54}$ of \$4.32

$=\$8$.

Hence, rate is 8%.

Principal on which the compound interest for the second year is

$$\$\{(1.08)^2-1.08\}=\$1;$$

$\therefore$ principal on which \$54 is the compound interest for the second year is $\$\frac{54}{.0864}$, or \$625.

39. If r is rate per cent.,

$$\text{then } 2500 \times \$\left(1+\frac{r}{100}\right)^2 = \$2756.25;$$

$$\therefore\ \$\left(1+\frac{r}{100}\right) = \$\sqrt{\frac{2756.25}{2500}};$$

$$\therefore\ \$\left(1+\frac{r}{100}\right) = \$\tfrac{525}{500} = \$1.05,$$

$$\text{and } \frac{r}{100} = .05;$$

$$\therefore\ r = 5\%.$$

$$\text{Amount in 4 yr.} = 2500 \times \$(1.05)^4$$
$$= \$3038.76\ldots$$

40. The number of miles constructed the fourth year is $\frac{6}{5}$ of $\frac{6}{5}$ of $\frac{6}{5}$ of the number of miles constructed the first year;

$\therefore$ $\frac{6}{5}$ of $\frac{6}{5}$ of $\frac{6}{5}$ of the number of miles constructed the first year
$= 432$ mi.;

$\therefore$ the number of miles constructed the first year
$= \frac{125}{216}$ of 432 mi. $=$ 250 mi.

EXERCISE XXXV. PAGE 182

1. Gain on $15\frac{3}{8}$ oz. $= \frac{5}{8}$ oz.;

$$\therefore\ \text{" " } 100 \text{ "} = \frac{100 \times \frac{5}{8}}{15\frac{3}{8}} \text{ oz.} = 4.065\ldots \text{ oz.};$$

$\therefore$ gain per cent. $= 4.065\ldots\%$.

5. If r represents the rate per cent.,

$$\text{then } 7600 \times \left(1+\frac{r}{100}\right) = 9196,$$

$$\text{and } \left(1+\frac{r}{100}\right)^2 = \tfrac{9196}{7600} = 1.21;$$

$$\therefore\ 1+\frac{r}{100} = \sqrt{1.21} = 1.1;$$

$$\therefore\ \frac{r}{100} = .1,$$

$$\text{and } r = 100 \times .1 = 10.$$

8. Sum of 10 results $= 10 \times 17.5 = 175$.

Sum of first 3 $= 3 \times 16.25 = 48.75$;

" next 4 $= 4 \times 16.5 = 66$;

$\therefore$ sum of last 3 $= 175 - 114.75 = 60.25$.

Ninth $=$ tenth $- 1$.

Eighth $=$ " $- 4$;

$\therefore$ sum of last 3 $= 3 \times$ tenth $- 5 = 60.25$;

$\therefore$ $3 \times$ tenth $= 65.25$;

$\therefore$ " $= 21.75$.

9. Since 6% of gross receipts $= \$42525$;

$\therefore$ the " " $= \$\dfrac{100 \times 42525}{6}$

$= \$708750$.

Now $3\frac{1}{2}$% of the capital $= \frac{54}{100}$ of $\$708750$;

$\therefore$ the paid-up capital $= \$\dfrac{100 \times 54 \times 708750}{3\frac{1}{2} \times 100}$

$= \$10935000$.

10. Part A does in 1 hr. $= \frac{1}{600}$.

" B " " 1 " $= \frac{3}{1600}$.

Time A and B take to do all $= \dfrac{1}{\frac{1}{600} + \frac{3}{1600}}$ hr.

$= \frac{4800}{17}$ hr.

Hence, A does $\frac{4800}{17} \times \frac{1}{600}$, or $\frac{8}{17}$ of the work,

and B " $\frac{4800}{17} \times \frac{3}{1600}$, " $\frac{9}{17}$ " it.

Cost of $\frac{17}{17}$ of work $= \$85$;

$\therefore$ " $\frac{8}{17}$ " $= \$(8 \times 5) = \$40$,

and " $\frac{9}{17}$ " $= \$(9 \times 5) = \$45$.

11. The cost of a policy of $\$100 = \$(5\frac{1}{4} + \frac{1}{5} + \frac{7}{16})$

$= \$5.8875$;

$\therefore$ policy which covers goods worth $\$94.1125$

$= \$100$;

$\therefore$ " " " " " $\$7905.45$

$= \$\dfrac{7905.45 \times 100}{94.1125}$

$= \$8400$.

13. $2\frac{1}{2}\% = \frac{10}{400}$ per unit.

$7\frac{1}{2}\% = \frac{30}{400}$ " "

$10\frac{1}{4}\% = \frac{41}{400}$ " "

The net increase $= \frac{10}{400}$ of males $+ \frac{10}{400}$ of females.

The decrease of males $= \frac{30}{400}$ "

The increase of females $= \frac{41}{400}$ which must

∴ = the decrease of males and the total net increase;

∴ $\frac{41}{400}$ of females $= \frac{10}{400}$ of males $+ \frac{10}{400}$ of females $+ \frac{30}{400}$ of males,

or $\frac{31}{400}$ of females $= \frac{40}{400}$ of males;

∴ the numbers are as 31 to 40.

15. Paper duty $= 1\frac{1}{2} \times 1\frac{1}{2}d. = 2\frac{1}{4}\,d.$

Cost of duty to retail dealer $= \frac{120}{100}$ of $\frac{110}{100}$ of $2\frac{1}{4}d. = 2.97d.$

20. Taking *B*'s flour as the standard and reducing *A*'s and *C*'s to this standard,

amount of flour *A* has of *B*'s standard $= \frac{110}{100}$ of 125 bbl.

$= 137.5$ bbl.

" " *C* " *B*'s "

$$= \frac{105\frac{5}{11}}{100} \text{ of } \frac{110}{100} \text{ of } 225 \text{ bbl.} = 261 \text{ bbl.}$$

Selling price of flour $= (125 + 150 + 225) \times \$7 = \$3500.$

Sum to be remitted $= \frac{96}{100}$ of $\$3500 = \$3360.$

He must pay \$3360 to *A*, *B*, and *C* in the proportion of 137.5, 150, and 261.

$$\text{Hence } A \text{ receives } \frac{137.5}{548.5} \text{ of } \$3360 = \$842.30 \text{ (nearly).}$$

$$B \quad \text{"} \quad \frac{150}{548.5} \text{ " } \$3360 = \$918.87 \quad \text{"}$$

$$C \quad \text{"} \quad \frac{261}{548.5} \text{ " } \$3360 = \$1598.83 \quad \text{"}$$

21. Sum gained, had none proved worthless $= \$1065.$

Cost of \$1 bill $= \$(.75 + .01\frac{1}{3}) = 76\frac{1}{3}$c.

Sum on which $\$.23\frac{2}{3}$ is gained $= \$1;$

$$\text{" " " } \$1065 \text{ " } = \$\frac{1065 \times 1}{.23\frac{2}{3}}$$

$= \$4500$

38. Conditional price $= \frac{90}{100}$ of selling price.
Amount of \$100 for 3 mo. $=$ \$$101\frac{1}{4}$;
$\therefore$ actual selling price 3 mo. before

$$= \frac{100}{101\frac{1}{4}} \text{ of } \frac{90}{100} \text{ of selling price}$$

$$= \frac{88\frac{8}{9}}{100} \text{ of selling price;}$$

$$\therefore \text{ discount allowed} = \frac{11\frac{1}{9}}{100} \text{ `` `` ``} = 11\tfrac{1}{9}\%.$$

Again, actual selling price 3 mo. after

$$= \frac{101\frac{1}{4}}{100} \text{ of } \frac{90}{100} \text{ of selling price}$$

$$= \frac{91\frac{1}{8}}{100} \text{ of selling price;}$$

$$\therefore \text{ discount allowed} = \frac{8\frac{7}{8}}{100} \text{ `` `` ``} = 8\tfrac{7}{8}\%.$$

39. Value of \$4 currency in gold $= \$\frac{4 \times 100}{118} = \$3\frac{23}{59}$.

$$\text{Gain on } \$3 \quad = \$\tfrac{23}{59};$$

$$\therefore \text{ `` `` } \$100 = \$\frac{100 \times \frac{23}{59}}{3} = \$12.99\ldots;$$

$$\therefore \text{ gain} = 12.99\%.$$

45. (*a*) The fixed charges must be referred to the same unit of time as the rest of the estimate, viz.: 1 wk.;

$$\therefore \text{ these} = \$\tfrac{250}{52} = \$\tfrac{125}{26}.$$

$$\text{Cost of 100 articles} = \$50 + \$\tfrac{125}{26} = \$54.8077;$$

$$\therefore \text{ cost of 1 article} = 0.548077.$$

(*b*) The profit on capital may be regarded as part of the cost of production. It would be so, in fact, if the money were borrowed at 30% interest. 30% added to \$0.548077 gives $\$\frac{130 \times .548077}{100}$.

Again, the commission is paid on the money actually received; to provide for it we must take $\frac{10}{9}$ of

$$\$\frac{130 \times .548077}{100}, \text{ or } \$\frac{10 \times 130 \times .548077}{9 \times 10}.$$

Next: 12% on bad debts means that 12 do not pay for 88 who do. To provide for it we must take $\frac{100}{88}$ of the selling price. The invoice price will therefore be

$$\$\frac{100 \times 10 \times 130 \times .548077}{88 \times 9 \times 100}, \text{ or } \$.899.$$

(*c*) To find the profit we must take $\frac{30}{100}$ of the cost price and multiply by 100×52.

$$\text{Annual profit} = \$\frac{30 \times 100 \times 52 \times .548077}{100} = \$855.$$

EXERCISE XXXVI. PAGE 194

12. Income from £1 in the 1st stock $= £\frac{6}{126} = £\frac{1}{21}$.
" " £1 " 2nd " $= £\frac{9}{210} = £\frac{3}{70}$.
Sum invested for difference of income of $£(\frac{1}{21} - \frac{3}{70}) = £1$;
∴ " " " " " " $£22\frac{1}{2} = £\dfrac{22\frac{1}{2} \times 1}{\frac{1}{21} - \frac{3}{70}}$
$= £4725.$

come from \$96 invested in 3 per cents. $= \$3$.
" " \$96 " " R. R. stocks $= \$\dfrac{96 \times 5}{100}$
$= \$4.80$;
∴ increase of income $= \dfrac{100 \times 1.80}{3}\% = 60\%$.

17. Net income on £91 invested $= £\left(3\frac{1}{2} - \dfrac{24\frac{1}{2}}{240}\right) = £\dfrac{1631}{480}$.
Sum invested for an income of $£\frac{1631}{480} = £91$;
∴ " " " " " $£\ 932 = £\dfrac{932 \times 91}{\frac{1631}{480}}$
$= £24960.$

18. Money from sale of £4500 stock $= £\dfrac{4500 \times 112.5}{100}$.
First income $= £\dfrac{4500 \times 5}{100} = £225$.
Second " $= £(225 + 168\frac{3}{4}) = £393\frac{3}{4}$.
Amount of Egyptian stock $= \dfrac{393\frac{3}{4} \times 100}{7} = £5625$.

Sum paid for £5625 stock $= £(45 \times 112.5)$;

∴ " " £100 " $= £\frac{100 \times 45 \times 112.5}{5625} = £90$.

23. On an investment of \$92 I lose \$7.

Income from investing $\$85 = \$\frac{85 \times 5}{100} = \$4\frac{1}{4}$;

∴ sum gained each year $= \$(4\frac{1}{4} - 3) = \$1\frac{1}{4}$;

hence, number of years $= \frac{7}{1\frac{1}{4}}$ yr. $= 5\frac{3}{5}$ yr.

25. Loss on each \$100 of stock bought $= \$2.875$.

He gained the dividend at $3\frac{1}{2}\% = \$3.50$;

∴ amount of stock from which the net profit is $\$.625 = \$100$;

∴ " " " " " $\$37.50 = \$\frac{37.50 \times 100}{.625}$

$= \$6000$.

27. Gross income $= \$\left(\frac{1277.25 \times 100}{98\frac{1}{4}} + 700\right) = \$2000$.

Income of \$6 is derived from an investment of \$120.

30. \$600 stock at 118 costs \$708.

Number of shares \$100 at $130 = \frac{708}{130 - 118} = 59$.

" " " " " $118 = 59 + 6 = 65$.

31. Income from $\$124.5 = \$5$;

∴ " " \$100 $= \$\frac{100 \times 5}{124.5} = \$4\frac{4}{249}$.

Income from $\$(\frac{100}{30} \times 34) = \$4\frac{5}{6}$;

∴ " " \$100 $= \$\frac{100 \times 4\frac{5}{6}}{\frac{100 \times 34}{30}} = \$4\frac{9}{34}$.

32. $3\frac{3}{4}\%$ of the capital

$= 3\%$ of (capital $- \$1200000) + 5\%$ of \$1200000;

∴ $\frac{3}{4}\%$ of the capital $= 5\%$ of $\$1200000 - 3\%$ of \$1200000.

35. Stock owned by $A = \frac{100}{5\frac{1}{4}}$ of \$1092 = \$20800.

Sum received for 250 shares $= \$\left(\frac{12500 \times 114\frac{1}{2}}{100}\right) = \$14312.50.$

" " " 166 " $= \$\left(\frac{8300 \times 117\frac{1}{4}}{100}\right) = \$9731.75;$

∴ sum received = \$24044.25.

37. Money realized by sale $= \$\frac{3430 \times 87\frac{1}{2}}{100} = \$\frac{343 \times 175}{20}.$

First income $= \$\frac{3430 \times 7}{200} = \$120.05.$

Second " $= \$\frac{343 \times 175 \times 4}{20 \times 98} = \$122.50;$

∴ the difference = \$2.45.

40. Amount of Turkish stock $= \$\frac{5000 \times 100}{80};$

∴ money from the sale of " $= \$\frac{5000 \times 100 \times 104}{80 \times 100} = \$6500.$

Income from \$90 invested in railway shares $= \$4\frac{1}{2};$

∴ new income $= \$\frac{6500 \times 4\frac{1}{2}}{90} = \$325.$

42. Present value $= \$\frac{2358 \times 100}{108}.$

Income from \$144 invested = \$9;

∴ " " $\$\frac{2358 \times 100}{108}$ " $= \$\frac{2358 \times 100 \times 9}{108 \times 144}$

$= \$136\frac{11}{24}.$

52. $\frac{105}{300}$ of his property + $\frac{103}{600}$ of his property + $\frac{102}{200}$ of his property = £6190.

53. Investment to give \$7 dividend = \$175;

∴ investment to give \$445.50 " $= \$\frac{445.50 \times 175}{7}$

$= \$11137.50;$

∴ selling price of flour $= \frac{100}{99}$ of \$11137.50;

∴ number of barrels $= \frac{1113750}{7.50 \times 99} = 1500.$

60. Suppose he sells 1 share of 3% stock at 89.

Income from investing in 4% stock $=\$\dfrac{4 \times 89}{92}$;

$\therefore\ \$(\frac{89}{23}-3)$ is difference in income when 1 share is sold.

61. See Art. 58.

62. The first payment of interest is $\$6$, and will be due in 1 yr.; its amount for 2 yr. will be $\$6(1.05)^2$; similarly, the amount of the second payment will be $\$6(1.05)$; and the amount of the third payment will be $\$6$. Hence, if P represents the present value of the bonds, we have

$$P(1.05)^3 = 100 + 6(1.05)^2 + 6(1.05) + 6 = 118.9150;$$

$$\therefore\ P = \frac{\$118.9150}{(1.05)^3} = \$102.723\ldots$$

63. Suppose he invests $\$99$ the first year,
then he invests $\$(99+6)$ the second year.
Income for the second year $=\$12\frac{36}{99}$;
$\therefore\ \$216\frac{36}{99}$ is amount when $\$99$ is invested each year.
$\$3606\frac{2}{33}$ " " $\$1650$.

65. Cost of $\$100$ stock $=\$76\frac{2}{3}$.
Selling price " " " " $=\$82\frac{3}{4}$;
$\therefore$ gain on " " " $=\$6\frac{1}{12}$.
Amount of stock to gain $\$6\frac{1}{12}$ $=\$100$;.
$\therefore$ " " " " " $\$121.66\frac{2}{3}=\$\dfrac{121.66\frac{2}{3}\times 100}{6\frac{1}{12}}=\$2000$;
$\therefore$ number of shares $=\frac{2000}{50}=40$.

68. Amount of stock bought $=\$\dfrac{16380 \times 100}{91}=\$18000$.

Money from $\$1200$ stock $=\$\dfrac{1200 \times 93.5}{100}=\$1122$.
" " $\$16800$ " $=\$\dfrac{16800 \times 85}{100}=\$14280$.
" " both sales $=\$15402$;
$\therefore$ loss $=\$978$.

$$\text{Original income} = \$\frac{18000 \times 3}{100} = \$540.$$

$$\text{New} \quad '' \quad = \$\frac{15402 \times 4.5}{102}$$

$$= \$679.50;$$

$$\text{hence, increase} = \$139.50.$$

EXERCISE XXXVII. PAGE 206

6. Representing B's share by 1,
A's " will be 3,
C's " " " 4;
∴ all the shares = 8 times B's "

7. When the second receives 8 apples, the first receives 7 and the third 10.

10. $2\,A\text{'s} + 2\,B\text{'s} + 2\,C\text{'s}$ do $\frac{46}{20}$ of work;
∴ $A + B + C$ " $\frac{23}{20}$ "
∴ A does $\frac{10}{20}$; B, $\frac{5}{20}$; and C, $\frac{8}{20}$ of work.

11. Weight of half crown $= \dfrac{2\frac{1}{2}}{66}$ of 5760 gr. $= \frac{7200}{33}$ gr.;
∴ number of grains of silver $= \frac{37}{40}$ of $\frac{7200}{33}$ gr. $= 201\frac{9}{11}$ gr.

13. Take the first part as the unit; then the second part will be $\frac{3}{5}$ of the first, and the third will be $\frac{3}{8}$ of the first.
Sum of the parts $= (1 + \frac{3}{5} + \frac{3}{8})$ of first $= \frac{79}{40}$ of first part.

19. $10s. + 5s. + 2\frac{1}{2}s. + 1s. + 6d. + 4d. = 232d.;$

$$\therefore \text{ number of each} = \frac{29 \times 20 \times 12}{232} = 30.$$

21. The pay of 7 women = the pay of 3 men.
" " " 14 boys = " " " $\frac{28}{5}$ women
= " " " $\frac{12}{5}$ men.
5 men + 3 men + $\frac{12}{5}$ men = $\frac{52}{5}$ men.

$$\text{Share of the men} = \frac{5}{\frac{52}{5}} \text{ of } \$10.40 = \$5.$$

22. Since there are 9 women, there must be 6 men and 15 children.

But the share of 9 women = share of 6 men,
and “ “ “ 15 children = “ “ 5 “

24. Take *B*'s share as the unit,
then *C*'s “ $= \frac{2}{5}$ of *B*'s + \$800,
then *A*'s “ $= \frac{5}{9}$ “ ($\frac{2}{5}$ of *B*'s + \$800) − \$300.
Sum of all the shares $= \frac{73}{45}$ “ *B*'s + \$944$\frac{4}{9}$.

26. Take the first fraction as the unit,
then the second $= \frac{22}{23}$ of the first,
and the third $= \frac{22}{24}$ “ “ “
Sum of the 3 fractions $= \frac{1586}{552}$ “ “ “
∴ $\frac{1586}{552}$ of the first $= \frac{183}{242}$.

28. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$;
∴ the boys over 15 get $\frac{1}{2}$ of \$400 = \$200,
and the boys between 10 and 15 get $\frac{1}{3}$ of \$400 = \$133$\frac{1}{3}$,
and the rest get $\frac{1}{6}$ of \$400 = \$66$\frac{2}{3}$.
Number of boys over 15 $= 200 \times 2 = 400$;
∴ “ “ “ in school $= 3 \times 400 = 1200$.

33. 2% of *A*'s capital = \$220;
∴ “ “ $= \$\dfrac{100 \times 220}{2}$ = \$11000;
∴ *B*'s and *C*'s “ $= \frac{3}{2}$ of \$11000 = \$16500.

34. The shares are in the ratio of $1 \times 4 \times 10$,
$2 \times 3 \times 12$, and $3 \times 1 \times 20$.

37. 20 men = 30 women, and 50 children = 30 women;
∴ sum received by (30 + 40 + 30) women
= \$600 for 1 week's work.

40. The ratio of costs is as $2 \times 9 \times 25 \times 12$ to $1 \times 8 \times 18 \times 10\frac{1}{2}$.

41. $\frac{99}{100}$ of a child's share $= \dfrac{2 \times 97}{100}$ of a brother's share;
∴ a brother's “ $= \frac{99}{194}$ of a child's share;
hence, 5 times a child's “ $+ \dfrac{3 \times 99}{194}$ times a child's share = \$12670.

43. Share of first $= \frac{30}{100}$ of \$300 = \$90.
" " second $= \frac{30}{100}$ " \$100 = \$30.

But as the machine works the same length of time for each and earns \$120 in all, or \$60 for each, therefore, the latter must give the former the difference between \$60 and \$30, or \$30.

44. Value of $10\frac{1}{2}$ oz. gold and $10\frac{1}{2}$ oz. silver = \$49 + \$(49 + 91)
= \$189.
Value of $10\frac{1}{2}$ oz. gold = \$168;
$\therefore$ " " $10\frac{1}{2}$ " silver = \$21.

48. 14 times first + 7 times second = 350;
$\therefore$ 2 " " + 1 " " = 50.
But 1 " " + 1 " " = 35;
$\therefore$ 1 " first = 50 − 35 = 15.

50. Since 1 lb. tea $= \frac{3}{2}$ lb. coffee $= \frac{9}{5}$ lb. cocoa;
$\therefore$ 1 " coffee $= \frac{6}{5}$ " cocoa,
and 1 " tea $= \frac{9}{5}$ " "

Quantity of each bought $= \frac{54 \times 112}{3}$ lb. = 2016 lb.
Hence, 2016 lb. coffee $= \frac{6}{5}$ of 2016 lb. cocoa,
and 2016 " tea $= \frac{9}{5}$ " " " "
and value of all $= (1 + \frac{6}{5} + \frac{9}{5}) \times 2016$ lb. cocoa;
$\therefore$ $= (672 \times 20 \times 12)d.$

51. Taking the rent of the top flat as the unit,
then the rent of the second flat from top $= \frac{3}{4}$ of rent of top,
and " " " " third " " " $= \frac{3}{4}$ of $\frac{7}{4}$ " " " "
$= \frac{21}{16}$ " " " "

53. C's share $= \frac{6}{7}$ of D's share.
B's share $= \frac{4}{5}$ of $\frac{6}{7}$ of D's " $= \frac{24}{35}$ " " "
A's " $= \frac{2}{3}$ " $\frac{24}{35}$ " " " $= \frac{16}{35}$ " " "
$\therefore$ $(1 + \frac{6}{7} + \frac{24}{35} + \frac{16}{35})$ " " " = \$21000.

54. Weight of nitre $= (\frac{75}{100} + \frac{77}{100})$ of 10 cwt. $= 15\frac{1}{5}$ cwt.

57. Amount of bread each eats $= \frac{8}{3}$ loaves;
$\therefore$ " " " given by first $= \frac{7}{3}$ "
and " " " " " second $= \frac{1}{3}$ "
$\therefore$ he pays the first 7 half-pence and the second 1 half-penny.

58. $\frac{1}{2}$ of the constituency vote for A;
$\frac{1}{4}$ " " " " " B;
$\frac{2}{3}$ of $\frac{1}{4}$ of the constituency vote for D and E;
now $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$;
∴ $\frac{1}{12}$ of the constituency $=540$, and
∴ number of electors $=6480$.

59. 15 men and 30 children get £(177 − 60), or £117;
∴ 1 man " 2 " " £$\frac{117}{15}$, or £7 16*s.*
But 1 " " 1 child " £6;
∴ 1 " " £1 16*s.*

60. $4285-(2540+980)=765$, the number of grains of soda and potash that take up 980 grains of the sulphuric acid;

$$\text{hence, } \frac{49\times\text{number of grains of soda}}{32}$$
$$+\frac{49\times(765-\text{number of grains of soda})}{48}=980;$$

or, $3\times$number of grains of soda $+\,2\times(765-$number of grains of soda$)=20\times96$;
∴ number of grains of soda $=1920-1530=390$;
∴ " " " " potash $=765-390=375$.

Exercise XXXVIII. Page 216

6. Sum to be divided between A and $B=\frac{57}{80}$ of \$29000.
A's share $=\frac{128}{228}$ of $\frac{57}{80}$ of \$29000 $=$ \$11600.

7. A's money was in the business 287 days, and B's 167 days.

$$\begin{aligned}287\times2400&=688800\\167\times1800&=\underline{300600}\\&\,989400;\end{aligned}$$

∴ A's share $=\frac{688800}{989400}$ of \$943
$=\$656\frac{820}{1649}$.

11. $2000+2800+1686+1014=7500$;
∴ D's share of capital $=\frac{1014}{7500}$ of \$45000
$=\$6084$.

12. *A* and *B*, and *A* and *C* contribute \$(1390 + 1500), or \$2890;
B and *C* contribute \$1590;

$\therefore$ twice *A*'s contribution = \$1300;

hence *A* contributes \$650, *B* \$740, *C* \$850, *D* \$960.

14. \$960 gains \$240 in 6 mo., or \$480 in 12 mo.;

$\therefore$ \$100 gains \$50 in 12 mo.

\$150 is amount of \$100 capital.
\$2400 " " " \$1600 "
\$50 is gained on \$100 in 12 mo.
\$400 " " " \$640 " $\frac{12}{1}\times\frac{100}{50}\times\frac{400}{640}$ mo.

16.
$$\left.\begin{array}{r}12\times 400=4800\\ 10\times 500=5000\end{array}\right\}\ 9800=A\text{'s capital for 1 mo.}$$
$$\left.\begin{array}{r}12\times 300=3600\\ 9\times 600=5400\end{array}\right\}\ \underline{9000=B\text{'s}\quad\text{"}\quad\text{"}\ \text{"}\ \text{"}}$$
$$18800.$$

A's share $=\frac{9800}{18800}$ of \$470 = \$245.

18.
$$\left.\begin{array}{r}4\times 1000=4000\\ 5\times 2500=12500\\ 3\times 2200=6600\end{array}\right\}\ 23100=A\text{'s capital for 1 mo.}$$
$$\left.\begin{array}{r}6\times 2500=15000\\ 5\times 3100=15500\\ 1\times 4100=4100\end{array}\right\}\ 34600=B\text{'s}\quad\text{"}\quad\text{"}\ \text{"}\ \text{"}$$
$$\left.\begin{array}{r}4\times 3000=12000\\ 4\times 1000=4000\\ 4\times500=2000\end{array}\right\}\ 18000=C\text{'s}\quad\text{"}\quad\text{"}\ \text{"}\ \text{"}$$

19.
$$\left.\begin{array}{l}1\tfrac{1}{2}\times\ 10=\ 15\text{ horses}=\ 45\text{ sheep}\\ 2\ \times\ 30=\ 60\text{ oxen }=120\quad\text{"}\\ 3\tfrac{1}{4}\times 100=325\text{ sheep}=325\quad\text{"}\end{array}\right\}=490.$$
$$\left.\begin{array}{l}2\tfrac{1}{2}\times\ 40=100\text{ horses}=300\text{ sheep}\\ 1\tfrac{1}{4}\times\ 50=62\tfrac{1}{2}\text{ oxen }=125\quad\text{"}\\ 3\ \times 115=345\text{ sheep}=345\quad\text{"}\end{array}\right\}=770.$$

22. *A* uses the *whole* house for 4 mo.; half of it for 5½ mo., and ⅓ of it for 2½ mo.

B uses ½ the house for 5½ mo., and ⅓ of it for 2½ mo.

C uses ⅓ the house for 2½ mo.

23. $\frac{2}{5}$ of profits is gained by $\$36000$ for 1 mo.
$\frac{1}{10}$ " " " " " $\frac{1}{4}$ of $\$36000$ " " "
or by $\$9000$ " " "
" " $\$2250$ " 4 "

Again, *C*'s profits $=\frac{5}{10}$ of profits;
$\therefore$ $\frac{}{10}$ of profits is gained by $\$45000$ for 1 mo.,
or by $\$5625$ " 8 "

EXERCISE XXXIX. PAGE 225

6.

$$£1=\frac{109\frac{5}{8}}{100} \text{ of } \$4\tfrac{4}{9}=\tfrac{877}{800} \text{ of } \$\tfrac{40}{9}.$$

$$\text{Cost of draft for } £1=\frac{100\frac{1}{4}}{100} \text{ of } \tfrac{877}{800} \text{ of } \$\tfrac{40}{9}.$$

7.

$$\text{Since } £1500=\$7300,$$
$$\therefore\ £1=\$4\tfrac{13}{15}.$$
$$\text{Now the advance on } \$4\tfrac{4}{9}=\$(4\tfrac{13}{15}-4\tfrac{4}{9})=\$\tfrac{19}{45};$$
$$\therefore\ \text{" " " } \$100=\$\frac{100\times\frac{19}{45}}{4\frac{4}{9}}$$
$$=\$9\tfrac{1}{2}.$$

Hence, exchange is at a premium of $9\frac{1}{2}\%$, and the quotation would be $109\frac{1}{2}$.

10. Interest on $\$1$ for 63 da. at $6\%=\$.0103561$;
$\therefore$ present worth of a draft for $\$1=\$.9896439$;
$\therefore$ cost of draft for $\$1=\$(.9896439+.0025)$
$=\$.9921439.$

14. Amount of gold in $\$1=\frac{9}{10}$ of $\frac{258}{10}$ gr. $=23.22$ gr.

$$\text{" " " " } £1=\tfrac{11}{12} \text{ of } \frac{40\times5760}{1869} \text{ gr.}$$
$$=\frac{11\times40\times480}{1869} \text{ gr.}$$

Now, 23.22 gr. $=\$1$;

$$\therefore\ \frac{11\times40\times480}{1869} \text{ gr.}=\$\frac{11\times40\times480\times1}{23.22\times1869}$$
$$=\$4.8665\ldots$$

15. Cost of exchange of \$2660 = \$2570.89;

$\therefore$ “ “ “ “ \$1 $= \$\dfrac{2570.89}{2660} = \$.9665.$

But the bank had the use of this money for 63 da., and allowed a deduction for interest.

Bank discount for 63 da. $= \$(.9665 \times \frac{63}{365} \times \frac{6}{100})$
$= \$.010009.$

Course of exchange $= \$.976509\ldots$

$1 - .9765\ldots = .0234;$

$\therefore$ exchange was at a discount of 2.34%.

20. Since $25\frac{1}{2}$ francs $= 11\frac{54}{60}$ florins,

$\therefore$ 20 francs $= \dfrac{20 \times 11\frac{54}{60}}{25\frac{1}{2}} = 9\frac{1}{3}$ florins = 9 florins 20 kreutzers.

25. The course of exchange is quoted by giving the value of 4 marks in cents.

Hence, the course of exchange is 4 marks = 95c.

30. Since 14 marcs banco = £1,

$\therefore$ 20 marcs banco, or 18 florins $= £\dfrac{20 \times 1}{14},$

and 28 florins, or 60 francs $= £\dfrac{28 \times 20 \times 1}{18 \times 14},$

4 francs, or 72c. $= £\dfrac{4 \times 28 \times 20 \times 1}{60 \times 18 \times 14};$

$\therefore$ \$1 $= £\dfrac{100 \times 4 \times 28 \times 20 \times 1}{72 \times 60 \times 18 \times 14} = £\frac{100}{486}.$

Hence, £1 = \$4.86.

33. $\dfrac{100\frac{1}{8}}{100}$ of bill = \$1513.89;

$\therefore$ “ $= \frac{800}{801}$ of \$1513.89 = \$1512.

Value of 6400 marks = \$1512.

39. Cost of draft by direct exchange = 20 × \$23.5 = \$470.

By London route 20.75 marks = \$4.875;

$\therefore$ “ “ “ 2000 “ $= \$\dfrac{2000 \times 4.875}{20.75}$
$= \$469.88.$

The circuitous is better by \$.12.

EXERCISE XL. PAGE 229

1. Cost of 1 lb. of the mixture $= \frac{87 \times 100}{116}$c. $= 75$c.

$\frac{11}{16}$ of the mixture consisted of the good tea and $\frac{5}{16}$ of it of the inferior kind.

Hence, $\frac{11}{16}$ of cost of dear tea $+ \frac{5}{16}$ of (cost of dear tea $-$ 12c.)
$= 75$c.;
$\therefore$ cost of dear tea $= (75 + \frac{60}{16})$c. $= 78\frac{3}{4}$c.,
and cost of cheap tea $= 66\frac{3}{4}$c.

4. Every gallon of the first mixture contains $\frac{12}{30}$ or $\frac{2}{5}$ gallons of wine, and every gallon of the second mixture contains $\frac{9}{12}$ or $\frac{3}{4}$ gallons of wine;

$\therefore$ $\frac{2}{5}$ of number from first $+ \frac{3}{4}$ of (14 $-$ number from first)
$= 7$ gal.;
$\therefore$ $(\frac{3}{4} - \frac{2}{5})$ of number from first $= (10\frac{1}{2} - 7)$ gal.,
and $\frac{7}{20}$ “ “ “ “ $= 3\frac{1}{2}$ gal.,
“ “ “ “ $= \frac{20 \times 3\frac{1}{2}}{7} = 10.$

Hence the number from the second $= 4$.

6. Interest for 18 da. at 6% $= \frac{216}{73000}$ of note.
Discount $= \frac{3}{200}$ “ “
$\therefore$ note $- (\frac{216}{73000} + \frac{3}{200})$ of note $=$ \$1190.234,
and $\frac{71689}{73000}$ “ “ $=$ \$1190.234.

7. A gain of \$120 in 6 mo. = a gain of \$20 in 1 mo.
“ “ “ \$150 “ 5 “ = “ “ “ \$30 “ “ “
“ “ “ \$210 “ 9 “ = “ “ “ $\$23\frac{1}{3}$ “ “ “
$\$73\frac{1}{3}$.

Sum from which $\$23\frac{1}{3}$ is gained = \$400;
$\therefore$ “ “ “ $\$73\frac{1}{3}$ “ “ $= \$\frac{73\frac{1}{3} \times 400}{23\frac{1}{3}} = \$1257\frac{1}{7}$.

8. After each drawing off, $\frac{3}{4}$ of the wine remaining in the cask is left.

Hence, the part finally left $= \frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of the wine
$= \frac{81}{256}$ of the wine.

10. Since 1 lira = \$0.22,

$\therefore$ 7500 lire = 7500 × \$0.22 = \$1650.

By circuitous exchange £1 = \$4.95;

$\therefore$ 26 francs = \$4.95,

and 1 lira, or $1\frac{1}{8}$ franc $= \$\dfrac{1\frac{1}{8} \times 4.95}{26}$;

$$\therefore 7500 \text{ lire} = \$\frac{7500 \times 9 \times 4.95}{8 \times 26} = \$1606.37\ldots$$

Hence the difference = \$43.63.

12. Cost price is $\frac{100}{116}$ of $14\frac{1}{2}$c., or $12\frac{1}{2}$c.

Diff.	$12\frac{1}{2}$		
$4\frac{1}{2}$	1 lb. at 8	1	= $14\frac{1}{4}$c. gain.
$9\frac{3}{4}$	3 " " $9\frac{1}{4}$	1	
...	....	...	
$\frac{1}{2}$	1 lb. at 13	$8\frac{1}{2} = 4\frac{1}{4}$	= $14\frac{1}{4}$c. loss.
$1\frac{1}{2}$	1 " " 14	$2\frac{2}{3} = 4$	
6	4 " " 14	1 = 6	

We have, therefore, 1 lb. at 8c.
$8\frac{1}{2}$ " " 13c.
and $2\frac{2}{3}$ " " 14c.

Of course, the above is only one of the many answers that might be found to this question.

16. Since the gain on \$2200 = \$880,

$$\therefore \text{ " " " } \$3500 = \$\frac{3500 \times 880}{2200} = \$1400.$$

But the gain for 2 mo. less = \$1120;

$\therefore$ 2 mo. gain on \$3500 = \$280.

17. Capital at end of first year

$= \frac{3}{2}$ of original capital − £1200.

Capital at end of second year

$= \frac{3}{2}$ of ($\frac{3}{2}$ of original capital − £1200) − £1200

$= \frac{9}{4}$ of original capital − £3000.

Capital at end of third year

$$=\tfrac{3}{2}\text{ of }(\tfrac{9}{4}\text{ of original capital} - £3000) - £1200$$

$$=\tfrac{27}{8}\text{ of original capital} - £5700.$$

Capital at end of fourth year

$$=\tfrac{3}{2}\text{ of }(\tfrac{27}{8}\text{ of original capital} - £5700) - £1200$$

$$=\tfrac{81}{16}\text{ of original capital} - £9750.$$

Hence, $\tfrac{81}{16}$ of original capital − £9750 = 4 × original capital.

21. By direct exchange \$100¾ = \$100 at New York;

$$\therefore\ \$14331.60 = \$\frac{14331.60 \times 100}{100\tfrac{3}{4}} = \$14224.91\ldots$$

$$\$1\text{ Cincinnati} = \$\left(\frac{1}{1.005}\right)\text{ St. Louis}$$

$$= \$\left(\frac{1}{.995} \times \frac{1}{1.005}\right)\text{ N. O.} = \$\left(\frac{1}{.99} \times \frac{1}{.995} \times \frac{1}{1.005}\right)\text{N. Y.};$$

$$\therefore\ \$14331.60\text{ Cincinnati} = \$\frac{14331.60}{.99 \times .995 \times 1.005}\text{ N. Y.}$$

$$= \$14476.72\ldots$$

Hence, gain = \$251.81.

24.

Diff's.	110		
	...		
30	80	1	1
25	85	1	2
10	100	1	2
..	...	.	..
5	115	13	20
		16	25

It is readily seen that 1 share at 80, 1 at 85, 1 at 100, and 13 at 115, would sell on the average at 110 without gain or loss, but these would make 16 shares not a submultiple of 100. Hence, we rearrange the quantities and find that 1 share at 80, 2 at 85, 2 at 100, and 20 at 115, will sell as required at 110. These make 25 shares, hence, 4 times these numbers must be taken, viz., 4 shares at 80, 8 at 85, 8 at 100, and 80 at 115.

25.

Diff's.	38		
6	32	1	1
2	36	1	7
..	..	..	..
4	42	2	5

It is observed that 1 lb. at 32c., 1 lb. at 36c. and 2 lb. at 42c., will make a mixture that will sell at 38c. But when 1 lb. at 32c. is taken there must be 7 lb. at 36c., and to balance this there must be 5 lb. at 42c. Hence, the mixture will contain (1 + 7 + 5) lb., or 13 lb

30. 80 gal., $87\frac{1}{2}\%$ strong, contains 70 gal. of alcohol.
This is to constitute $66\frac{2}{3}\%$, or $\frac{2}{3}$ of the new mixture.
70 gal. is $\frac{2}{3}$ of 105 gal.;
∴ the mixture contains 105 gal.,
and the water added $=(105-80)$ gal. $=25$ gal.

31. 22 lb. of coffee and 22 lb. of chicory cost $8.80.
But 22 " " " $1.10;
∴ 22 " coffee " $7.70;
∴ 1 " " " 35c.

34. 5 oz. 18 carats fine $=$ 90 oz. 1 carat fine.
7 " 15 " " $=105$ " 1 " "
mixture of 12 oz. $=195$ " 1 " "
∴ fineness of mixture $=\frac{195}{12}$ carats $=16\frac{1}{4}$ carats.

Hence, weight of pure gold $=\dfrac{16\frac{1}{4}}{24}$ of 4 oz. $=2\frac{17}{24}$ oz.

Exercise XLI. Page 234

10. The reciprocal ratio is expressed by inverting the order of the terms; hence, the reciprocal ratio of $\frac{1}{3}:\frac{1}{6}$ is $\frac{1}{6}:\frac{1}{3}$, or $\frac{1}{6}\div\frac{1}{3}$. $\frac{1}{6}:\frac{1}{3}=\frac{1}{2}$.

Again, the reciprocal ratio of $2\frac{1}{3}:7.\dot{7}$ is $7.\dot{7}:2\frac{1}{3}$, or $7\frac{7}{9}\div2\frac{1}{3}$. $7\frac{7}{9}\div2\frac{1}{3}=3\frac{1}{3}$.

13. If $\frac{2}{3}$ be the given ratio, then adding any number, say 5, to each of the terms, we have $\dfrac{2+5}{3+5}=\frac{7}{8}$. Comparing this ratio with $\frac{2}{3}$, we have $\frac{16}{24}$ and $\frac{21}{24}$. Hence, we see that this ratio is increased by adding the same number to each of its terms.

Again, if we take $\frac{4}{3}$ as the ratio and add, say 2, to each of the terms, we have $\dfrac{4+2}{3+2}=\frac{6}{5}$. Comparing this with the original ratio, we have $\frac{20}{15}$ and $\frac{18}{15}$. Hence, we see that this ratio is diminished by adding the same number to each of its terms. A ratio is, therefore, increased or diminished by adding the same number to each of its terms according as the antecedent is less or greater than its consequent.

14. When B is supposed to win, B runs 1760 yd. while A runs 1749 yd.;

$\therefore$ B's rate : A's rate as 1760 : 1749, or as 160 : 159.

But 160 represents $\frac{4}{3}$ of B's actual rate;

$\therefore$ B's actual rate is $\frac{3}{4}$ of 160, or 120.

Hence, A's rate : B's rate as 159 : 120.

EXERCISE XLII. PAGE 237

11. The proportion is 30 ft. : $12\frac{1}{2}$ ft. :: \$$x$: \$650;

$$\therefore \$x = \$\frac{30 \times 650}{12\frac{1}{2}} = \$1560.$$

20. 84 ft. : $2\frac{1}{4}$ ft. :: 125 ft. : height required;

$$\therefore \text{ height required} = \frac{2\frac{1}{4} \times 125}{84} \text{ ft.} = 3\tfrac{39}{112} \text{ ft.}$$

31. The 5000 men have provisions for 90 da. remaining.

Hence, 4500 men : 5000 men :: 90 da. : time required;

$$\therefore \text{ time required} = \frac{5000 \times 90}{4500} \text{ da.} = 100 \text{ da.}$$

Hence, they will last 10 da. longer.

34. Gain in $6\frac{1}{2}$ rounds $= \frac{1}{3}$ mi.

$6\frac{1}{2}$ rounds : 9 rounds :: $\frac{1}{3}$ mi : A's gain;

$$\therefore A\text{'s gain} = \frac{9 \times \frac{1}{3}}{6\frac{1}{2}} \text{ mi.} = \tfrac{6}{13} \text{ mi.}$$

35. The hands of the watch will be together for the fourth time after noon at $16\frac{4}{11}$ min. past 3.

The watch will have gone $(6\frac{7}{11} + 180 + 16\frac{4}{11})$ min., or 203 min.

But $59\frac{17}{41}$ min. on the watch correspond to 60 min. of true time;

$\therefore$ $59\frac{17}{41}$: 203 :: 60 min. : time required;

$$\therefore \text{ time required} = \frac{203 \times 60}{59\frac{17}{41}} \text{ min.} = 205 \text{ min.} = 3 \text{ hr. } 25 \text{ min.}$$

EXERCISE XLIII. PAGE 244

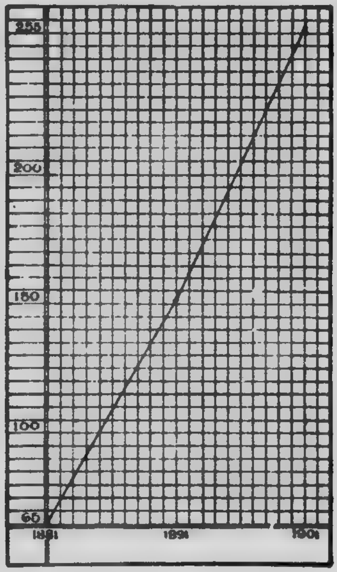
1. Draw two lines, one 5 dm. and the other 8 dm. in length. The former will represent the kilometre and the latter the mile.

2. Draw two lines, one 5 dm. and the other 11 dm. in length. The former will represent the pound, and the latter the kilogram.

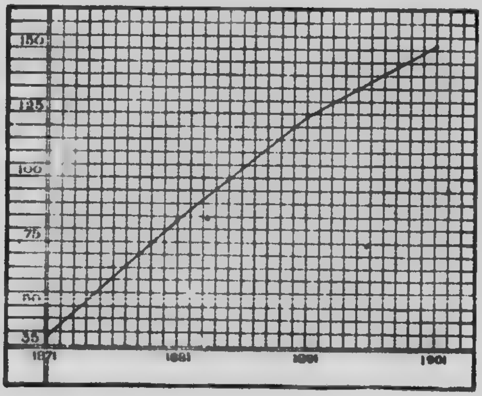
3. Draw two lines, one 25 cm. and the other 44 cm. in length. Since 25 l. = 44 pt., the latter will represent the litre and the former the pint.

4. Draw two lines, one 37 cm. and the other 10 cm. in length. The former will represent the cord and the latter the stere.

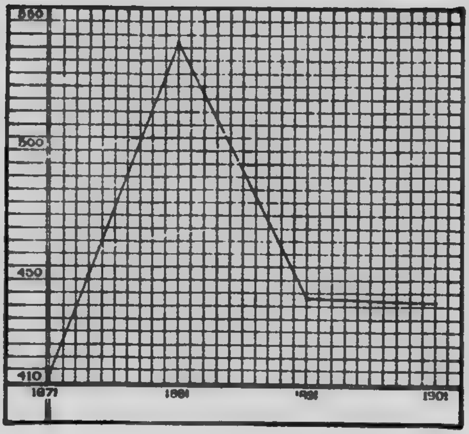
5.



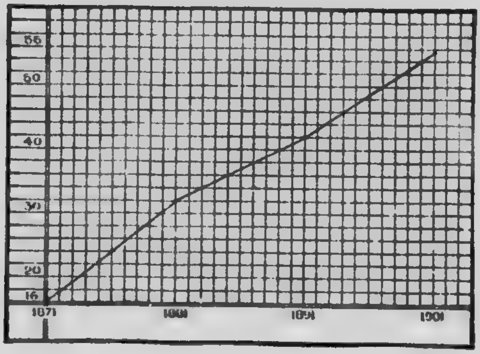
6.



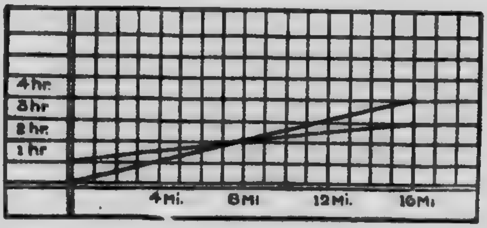
7.



8.

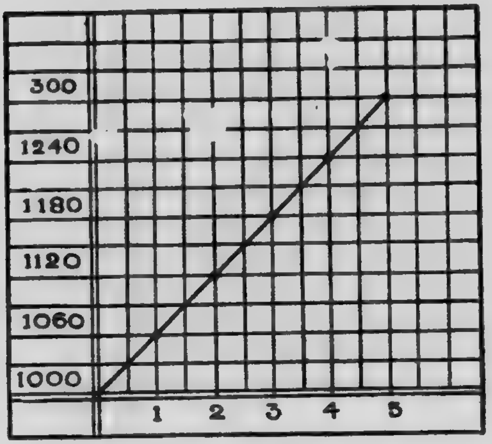


10.

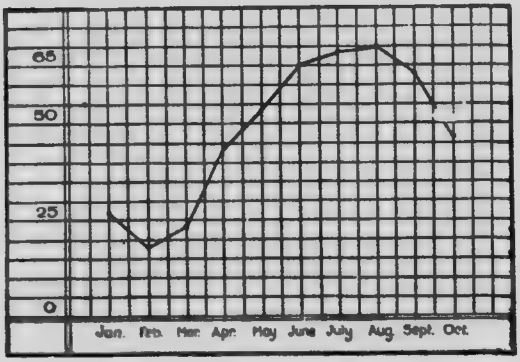


The graph shows that Jones' rate is 8 mi. per hour and that he overtakes Smith at the end of 8 mi.

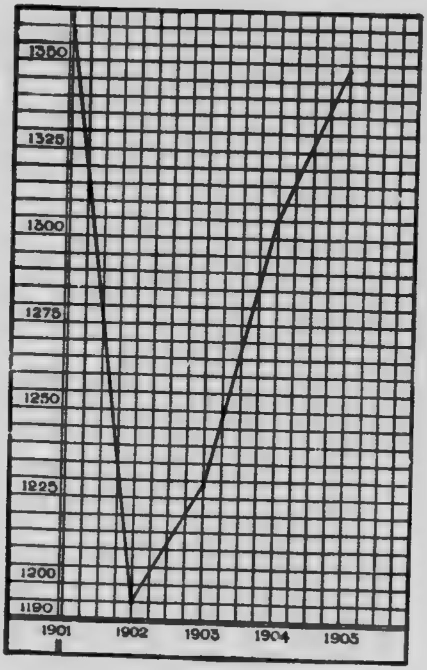
11.


300
1240
1180
1120
1060
1000
1
2
3
4
5

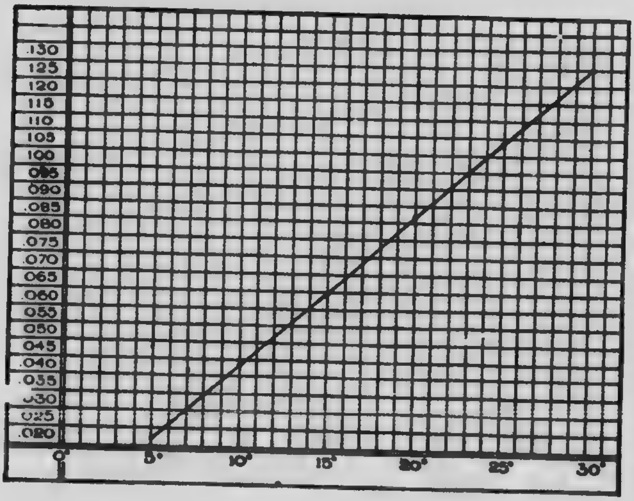
12.


65
50
25
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.

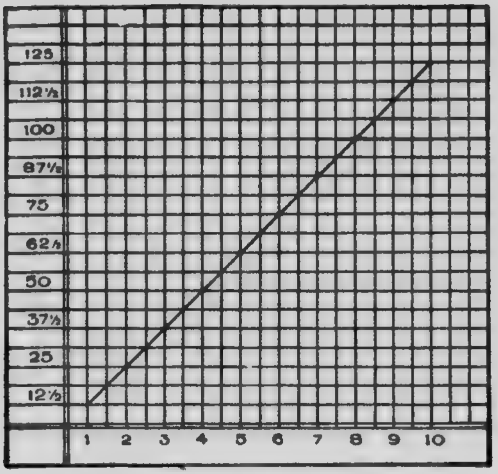
13.


1350
1325
1300
1275
1250
1225
1200
1190
1901
1902
1903
1904
1905

14.

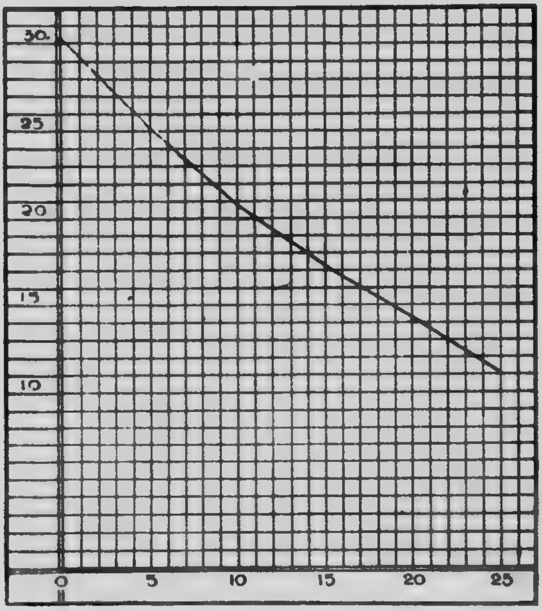

.130
125
120
115
110
105
100
.095
.090
.085
.080
.075
.070
.065
.060
.055
.050
.045
.040
.035
.030
.025
.020
0°
5°
10°
15°
20°
25°
30°

15.



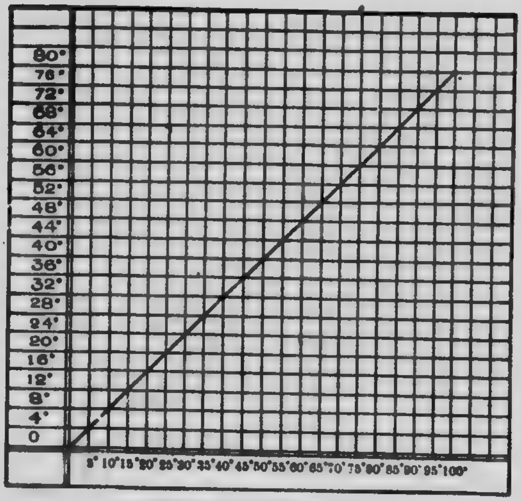
NUMBER OF YARDS.

16.



From the graph it will be seen that the barometer will be 27.1 in. high at 3000 ft.; 28.2 in. at 2000 ft. and 22.5 at 8000 ft.

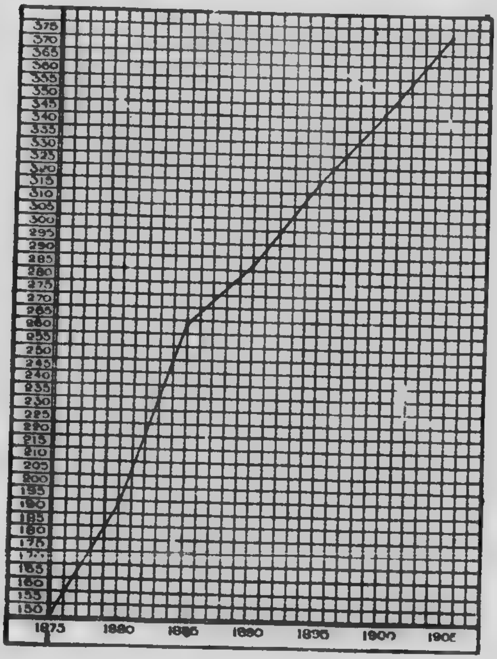
18.



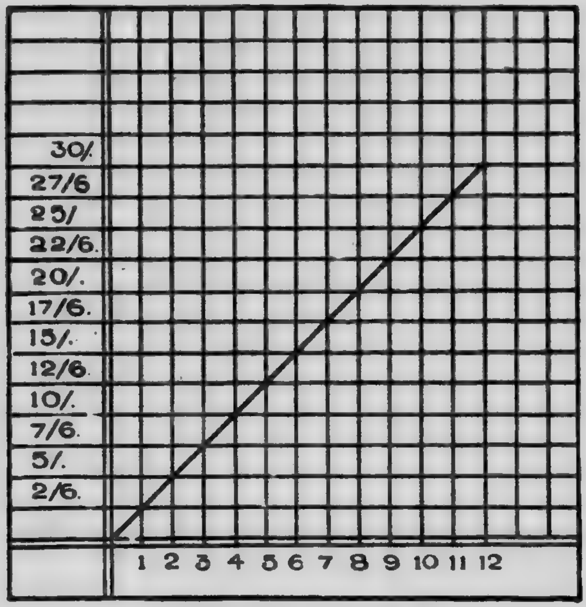
DEGREES CENTIGRADE.

It is easily seen that 10 R. is equal to 12½° C., and that 15° C. is equal to 12° R.

19.

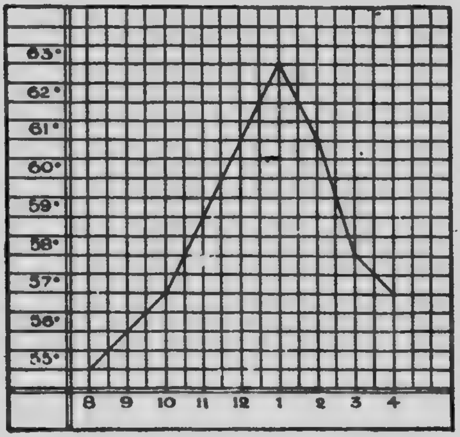


21.



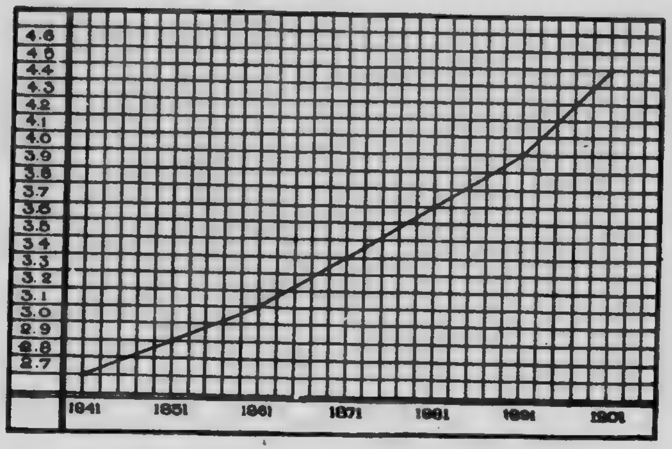
NUMBER OF POUNDS.

22.



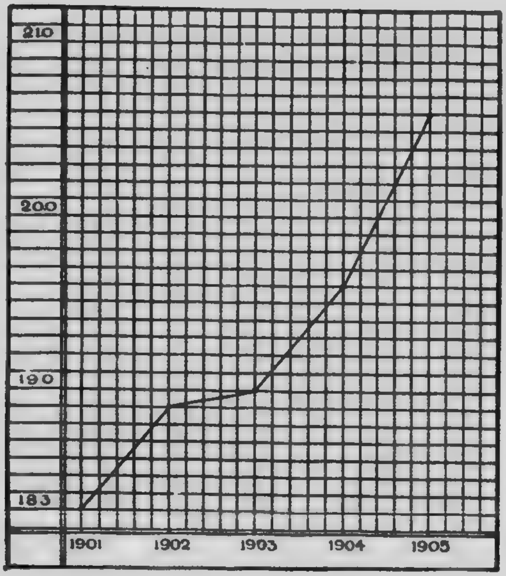
The following is the graph for Scotland :—

23.



(*c*) Draw the graph for the combined populations.

24. (*a*)



EXERCISE XLIV. PAGE 251

5. Side of square $= \frac{160}{4}$ in. $= 40$ in.
Sides of rectangle $= \frac{2}{5}$ of 80 in. and $\frac{3}{5}$ of 80 in.
Area of square $= 40^2$ sq. in. $= 1600$ sq. in.
Area of rectangle $= (32 \times 48)$ " " $= 1536$ " "
$\therefore$ difference of area $= 64$ sq. in.

EXERCISE XLVI. PAGE 254

6. Perimeter of kitchen $= 68$ ft.
" " dining-room $= 68$ ft.
" " parlor $= 74$ ft.
" " hall $= 54$ ft.
Area of walls $= (264 \times 11)$ sq. ft.
$= 2904$ sq. ft.
Area of ceilings $= (16 \times 18 + 16 \times 15 + 3 \times 9 + 14 \times 20 + 3 \times 12 + 13 \times 14)$ sq. ft. $= 1053$ sq. ft.

$$\text{Total area to be plastered} = \frac{2904 + 1053}{9} \text{ sq. yd.}$$

$= 439\frac{2}{3}$ sq. yd.
Cost of plastering $= 440 \times 42$c.
$= \$184.80$.

EXERCISE XLVII. PAGE 256

3. (*a*) Area of kitchen and hall $= (18 \times 16 + 13 \times 14)$ sq. ft.
$= 470$ sq. ft.
Deduction for stairways $= (4 \times 8 + 4 \times 6\frac{2}{5})$ sq. ft.
$= 57\frac{3}{5}$ sq. ft.

$$\text{Number of square yards of linoleum} = \frac{412\frac{2}{5}}{9} = \frac{2062}{45}.$$

Cost of linoleum $= \frac{2062}{45} \times \$1.25 = \$57\frac{5}{18}$.

EXERCISE LII. PAGE 263

4. Distance of top from ground $= \sqrt{(41^2 - 9^2)}$ ft. $= 40$ ft.
$\therefore$ distance the top will drop $= (41 - 40)$ " $= 1$ ft.

11. Let d = distance from the ground.

Then $(98-d)^2 = d^2 + 84^2$;

$\therefore$ $9604 - 196d + d^2 = d^2 + 7056$;

$\therefore$ $196d = 2548$;

$\therefore$ $d = \frac{2548}{196} = 13$;

$\therefore$ The tree broke 13 ft. from the ground.

EXERCISE LIII. PAGE 266

When the triangle is a right-angled one the length of the perpendicular dropped on the hypothenuse is most readily found as follows:

Since twice the measure of the area of the triangle is equal to the product of the measures of the sides containing the right angle and is also equal to the product of the measures of the perpendicular on the hypothenuse and the hypothenuse, therefore, to find the measure of this perpendicular divide the product of the measures of the sides containing the right angle by the measure of the hypothenuse.

EXERCISE LIV. PAGE 267

2. The quadrilateral is divided by the diagonal into two triangles.

4. The side A D is (733 − 325) ft. longer than B C.

733 ft. − 325 ft. = 408 ft.

$\frac{408}{2}$ ft. = 204 "

Width of A B C D = $\sqrt{(325^2 - 204^2)}$ ft.

= 253 ft.

Area of A B C D = $\left(\frac{733+325}{2} \times 253\right)$ sq. ft.

= 133837 sq. ft.

EXERCISE LVI. PAGE 270

7. The diagonal of the square is the diameter of the circle. Now diameter of circle = $\frac{7}{22}$ of 88 in. = 28 in.

Hence, 28 in. is the hypothenuse of a right-angled triangle, the other sides being the sides of the square;

hence, $2 \times (\text{measure of side})^2 = 28^2$;

$\therefore$ $(\text{measure of side})^2 = \frac{784}{2} = 392$;

$\therefore$ $\text{measure of side} = \sqrt{392}$

$= 19.79$;

$\therefore$ side of square $= 19.79$ in.

8. Radius of outer circle $= \frac{1}{2}$ of $\frac{7}{22}$ of 880 yd. $= 140$ yd.

" " inner " $= \frac{1}{2}$ " $\frac{7}{22}$ " 792 " $= 126$ "

$\therefore$ width of road $= 14$ yd.

Area of outer circle $= (\frac{22}{7} \times 140^2)$ sq. yd.

" " inner " $= (\frac{22}{7} \times 126^2)$ " "

$\therefore$ area of road $= (\frac{22}{7} \times 266 \times 14)$ sq. yd.

$= 11704$ sq. yd.

EXERCISE LVII. PAGE 272

4. Radius of circle $= \sqrt{(\frac{7}{22} \text{ of } 616)}$ ft. $= 14$ ft.

Now, measure of length of arc $\times 14 = 2 \times 56$;

$\therefore$ measure of length of arc $= \dfrac{2 \times 56}{14} = 8$;

$\therefore$ " " " $= 8$ ft.

EXERCISE LVIII. PAGE 273

10. Draw the figure and let the produced sides meet in a, and let the distance between a and the shorter parallel side be x and y, respectively; then there are two similar triangles formed and

$18 : 16 + x :: 10\frac{1}{2} : x$ (1);

also, $18 : 12 + y :: 10\frac{1}{2} : y$ (2).

Solving (1), x is found to be $22\frac{2}{5}$, and

" (2), y " " $16\frac{4}{5}$;

$\therefore$ distance required $= (22\frac{2}{5} + 16)$ ft. $= 38\frac{2}{5}$ ft.

and " " $= (12 + 16\frac{4}{5})$ " $= 28\frac{4}{5}$ "

13. The diameters of the pipes are as $2 : 1$.

The areas of the bores " " " $4 : 1$.

Area of a pipe to do as much work as the two would be represented by 5 in (a) and 3 in (b).

Now, time for a pipe whose bore is represented by 4 to fill the cistern = 20 min.;

∴ " " " " " represented by 5 to fill the cistern $= \dfrac{4 \times 20}{5}$ min.

$= 16$ min.,

and " " " " " represented by 3 to fill the cistern $= \dfrac{4 \times 20}{3}$ min.

$26\frac{2}{3}$ min.

Exercise LIX. Page 275

6. $$\text{Breadth} = \frac{\frac{5}{11} \text{ of the area of the walls}}{21}$$
$$= \frac{\frac{5}{11}\{(42 + 2 \times \text{breadth}) \times 10\frac{1}{2}\}}{21}$$
$$= \tfrac{5}{11}\{21 + \text{breadth}\}$$
$$\therefore \tfrac{6}{11} \text{ of breadth} = \tfrac{105}{11} \text{ ft.};$$
$$\therefore \text{breadth} = \tfrac{11}{6} \text{ of } \tfrac{105}{11} \text{ ft.} = 17\tfrac{1}{2} \text{ ft.}$$

13. $$\text{Area of walls} = \{(2 \times \text{length} + 2 \times \text{breadth}) \times 11\} \text{ sq. ft.}$$
$$= \{(4 \times \text{breadth} + 2 \times \text{breadth}) \times 11\} \text{ sq. ft.}$$
$$= (66 \times \text{breadth}) \text{ sq. ft.};$$
$$\therefore 66 \times \text{breadth} = (143 \times 3) \times 2,$$
$$\text{and breadth} = \frac{143 \times 6}{66} \text{ ft.} = 13 \text{ ft.},$$
$$\text{and length} = 26 \text{ ft.};$$
$$\therefore \text{length of moulding} = \frac{2 \times 26 + 2 \times 13}{3} \text{ ft.} = 26 \text{ yd.}$$

19. The diagonal is 20.

If the foot of one of the perpendiculars is x units from the angle, then
$$16^2 - (20 - x)^2 = 12^2 - x^2;$$
$$\therefore 40x = 288,$$
$$\text{and } x = 7.2.$$

Hence, distance between the feet of the perpendiculars
$$(20 - 2 \times 7.2) \text{ ft., or } 5.6 \text{ ft.}$$

30. Length of diagonal of floor $= \sqrt{(18^2+13^2)}$ ft.
" " " " room $= \sqrt{[\{\sqrt{(18^2+13^2)}\}^2+9^2]}$ ft.
$= \sqrt{(18^2+13^2+9^2)}$ ft.
$= 23.95 \ldots$ ft.

31. By drawing the figure it will be seen that there are two similar triangles, hence,

$6 : 7\frac{1}{2} :: 15 :$ (measure of distance $+ 7\frac{1}{2}$);
$\therefore$ $6 \times$ measure of distance $+ 6 \times 7\frac{1}{2} = 15 \times 7\frac{1}{2}$;
$\therefore$ $6 \times$ measure of distance $= 9 \times 7\frac{1}{2}$;
$\therefore$ " " " $= \dfrac{9 \times 7\frac{1}{2}}{6} = 11\frac{1}{4}$;
$\therefore$ distance $= 11\frac{1}{4}$ ft.

47. Draw the figure and it will be seen that the free end of the string describes (1) an arc of 120° of a circle 14 in. in diameter, (2) an arc of 120° of a circle 28 in. in diameter, (3) an arc of 120° of a circle 42 in. in diameter.

Hence, distance moved $= (\frac{1}{3}$ of $\frac{22}{7}$ of $14)$ in. $+ (\frac{1}{3}$ of $\frac{22}{7}$ of $28)$ in. $+ (\frac{1}{3}$ of $\frac{22}{7}$ of $42)$ in. $= 88$ in.

EXERCISE LXI. PAGE 286

8. Area of surface of ring $= (\frac{22}{7} \times 33^2 - \frac{22}{7} \times 30^2)$ sq. in.
$= (\frac{22}{7} \times 63 \times 3)$ sq. in.
$= 594$ sq. in.;
$\therefore$ volume of ring $= (2 \times 594)$ cu. in.
$= 1188$ cu. in.

14. Cubic content of vessel $= (4 \times 277.274)$ cu. in.
If the radius is r, then the cubic content in terms of
r is $\frac{22}{7} \times r^2 \times 2r$;
$\therefore$ $\frac{22}{7} \times r^2 \times 2r = 4 \times 277.274$;
$\therefore$ $r = \sqrt[3]{\left(\dfrac{4 \times 277.274}{\frac{22}{7} \times 2}\right)}$ in.
$= 5.6$ in., nearly;
$\therefore$ depth $= 2 \times 5.6$ in., nearly
$= 11.2$ in.

EXERCISE LXII. PAGE 288

4. Perimeter of base $= 5 \times 18$ in. $= 90$ in.

Area of lateral surface $= \dfrac{7\frac{1}{2} \times 10}{2}$ sq. ft. $= 37\frac{1}{2}$ sq. ft.

EXERCISE LXIII. PAGE 289

6. Altitude of pyramid $= \sqrt{(61^2 - 11^2)}$ ft. $= 60$ ft.

Volume " " $= \dfrac{60 \times 22 \times 22}{3}$ cu. ft.

$= 9680$ cu. ft.

EXERCISE LXIV. PAGE 291

5. Radius of cone $= \frac{1}{2}$ of $\frac{7}{22}$ of 342 in. $= \frac{1197}{22}$ in.

Slant height $= \sqrt{\{(252)^2 + (\frac{1197}{22})^2\}}$ in. $= 257.806$ in.

Lateral surface $= (\frac{342}{2} \times 257.806)$ sq. in. $= 44084.826$ sq. in.

8. Slant height of vessel $= \sqrt{(70^2 + 24^2)}$ in. $= 74$ in.

Area of lateral surface $= (74 \times \frac{22}{7} \times \frac{48}{2})$ sq. in.

$= \frac{39072}{7}$ sq. in.

Area of lid $= (\frac{22}{7} \times 24 \times 24)$ sq. in.

$= \frac{12672}{7}$ sq. in.;

$\therefore$ number of square feet of iron $= \left(\dfrac{39072 + 12672}{7 \times 144}\right) = 51\frac{1}{3}$.

EXERCISE LXVI. PAGE 294

8. Since 12 in. represents 8000 mi.,

1 " " $\frac{8000}{12}$ mi., or $666\frac{2}{3}$ mi.

EXERCISE LXVII. PAGE 295

2. Cubic content of block $= 1$ cu. ft.

" " " sphere $= (\frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ cu. ft.

$= \frac{11}{21}$ cu. ft.;

$\therefore$ part cut away $= \frac{10}{21}$ cu. ft.

$= 822\frac{6}{7}$ cu. in.

6. Cubic content of sphere $= (\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6)$ cu. in.

“ “ “ cylinder $= 8 \times \{\frac{22}{7} \times (6\frac{1}{2})^2 - \frac{22}{7}r^2\}$

$= 8 \times \frac{22}{7}\{(6\frac{1}{2})^2 - r^2\};$

$\therefore\ 8 \times \frac{22}{7}\{(6\frac{1}{2})^2 - r^2\} = \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6;$

$$(6\tfrac{1}{2})^2 - r^2 = \frac{\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6}{8 \times \frac{22}{7}} = 36;$$

$\therefore\ r^2 = 6\frac{1}{4};$

$\therefore\ r = \frac{5}{2} = 2\frac{1}{2}.$

Hence, the thickness of the iron $= (6\frac{1}{2} - 2\frac{1}{2})$ in. $= 4$ in.

7. Let the volume of each be 1 cu. ft.

Surface of cube $= 6 \times 1$ sq. ft.

$= 6$ sq. ft.;

Also, $\frac{4}{3} \times \frac{22}{7} \times r^3 = 1;$

$\therefore\ r = \sqrt[3]{(\frac{3}{4} \text{ of } \frac{7}{22} \times 1)}$ ft.;

$\therefore$ surface of sphere $= \{4 \times \frac{22}{7}(\sqrt[3]{\frac{21}{88}})^2\}$ sq. ft.

$= \{4 \times \frac{22}{7} \times \frac{1}{88} \times \sqrt[3]{21^2 \times 88}\}$ sq. ft.

$= (\frac{1}{7} \times 33.853 \ldots)$ sq. ft.

$= 4.833 ..$ sq. ft.;

$\therefore$ surface of cube : surface of sphere :: 6 : 4.833...

:: 1 : .805...

11. Surface of the lead ball $= (\frac{22}{7} \times 4 \times 4)$ sq. in.;

$\therefore$ “ “ “ silver “ $= (2 \times \frac{22}{7} \times 4 \times 4)$ sq. in.;

$\therefore$ diameter “ “ “ “ $= \sqrt{(\frac{7}{22} \times 2 \times \frac{22}{7} \times 4 \times 4)}$ in.

$= 5.6568 \ldots$ in.;

$$\therefore \text{ thickness of the silver} = \left(\frac{5.6568 - 4}{2}\right)$$

$= .8284$ in.

13. Cubic content of ball $= (\frac{1}{6} \text{ of } \frac{22}{7} \text{ of } 27)$ cu. in.

$= \frac{99}{7}$ cu. in.

Area of base of vessel $= (3\frac{1}{7} \times 3^2)$ sq. in.

$= \frac{198}{7}$ sq. in.

Height to which the water rises $= (\frac{99}{7} \div \frac{198}{7})$ in.

$= \frac{1}{2}$ in.

Hence, the water will be $4\frac{1}{2}$ in. deep in the vessel.

EXERCISE LXVIII. PAGE 298

5. Cubic content $=\{\frac{1}{3}\times 25\times(\frac{77}{2}+\frac{77}{128}+\frac{77}{16})\}$ cu. m.
$=365\frac{365}{384}$ cu. m.

6. Area of section of drain $=\left(\frac{25+16}{2}\times 4\right)$ sq. ft.
$=82$ sq. ft.

Distance water flows in 60 min. = 2640 ft.;
∴ " " " " 10 " $=\frac{1}{6}$ of 2640 ft. = 440 ft.
Cubical content of drain in 440 ft. = (440×82) cu. ft.
Now, 1 cu. ft. of water $=6\frac{1}{4}$ gal.;
∴ (440×82) " " " " $=440\times 82\times 6\frac{1}{4}$ gal.
$=225500$ gal.

8. Altitude of frustum $=\sqrt{(5^2-3^2)}$ ft. = 4 ft.

EXERCISE LXIX. PAGE 300

3. If d and d' represent the diameters of a 25c. piece and a 5c. piece, respectively, then the areas of the circular faces are as d^2 to $(d')^2$, and the quantity of silver in each as $7d^2$ to $5(d')^2$;

$$\therefore\ 7d^2=5\times 5(d')^2;$$
$$\therefore\ d\sqrt{7}=5d';$$
$$\therefore\ d : d' :: 5 : \sqrt{7}.$$

4. Volume of first : 3 × volume of first :: 20^3 : (measure of breadth of second)^3;
∴ (measure of breadth of second)$^3=3\times 20^3$;
∴ measure of breadth of second $=20\sqrt[3]{3}$
$=28.845$;
∴ breadth of second = 28.845 ft.

10. 27 : 125 :: 1^3 : (measure of dimension)^3;
∴ measure of dimension $=\frac{5}{3}$.
Now area of first : area of second :: $1^2 : (\frac{5}{3})^2$;
∴ area of second $=\frac{25}{9}\times$ area of first.
Again, cost of first = \$3.60;
∴ " " second $=\frac{25}{9}\times$ \$3.60 = \$10.00.

13. Volume of first plate $= (\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 1)$ cu. in.
$= 38\frac{1}{2}$ cu. in.
" " second " $= (\frac{22}{7} \times 7 \times 7 \times 1)$ cu. in.
$= 154$ cu. in.
Volume of both plates $= 192\frac{1}{2}$ cu. in.;
$\therefore$ area of circular plate $= \dfrac{192\frac{1}{2}}{5}$ sq. in. $= 38.5$ sq. in.
Now, the radius of plate $= \sqrt{(\frac{7}{22} \text{ of } 38.5)}$ in. $= \frac{7}{2}$ in.;
$\therefore$ diameter of plate $= 7$ in.

EXERCISE LXX. PAGE 301

3. Measure of diagonal of cube = measure of edge $\times \sqrt{3} = 6$;
$\therefore$ measure of edge $\times \sqrt{3} = 6$;
$\therefore$ measure of edge $= \dfrac{6}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$;
$\therefore$ volume of cube $= (2\sqrt{3})^3$ cu. ft. $= 41.568$ cu. ft.

13. The volumes will vary as the areas of the similar surfaces;
$\therefore$ area of first plate : area of part cut away :: 24^2 : (diameter of hole)^2;
$\therefore 1 : \frac{1}{4} :: 24^2$: (diameter of hole)^2;
$\therefore$ diameter of hole $= \sqrt{(\frac{1}{4} \times 24^2)}$ in. $= 12$ in.

17. Length $= 3 \times$ width;
$\therefore$ measure of solidity
$= \frac{16}{3} \times (3 \times$ measure of width$) \times$ measure of width;
$\therefore \frac{16}{3} \times 3 \times ($measure of width$)^2 = 100$;
$\therefore$ measure of width $= \sqrt{(\frac{100}{16})} = \frac{10}{4}$;
$\therefore$ width $= 2\frac{1}{2}$ ft.,
and length $= 7\frac{1}{2}$ ft.

19. Volume of bottom $= 1 \times (72 \times 42 \times 1)$ cu. in.
$= 3024$ cu. in.
" " sides $= 2 \times (47 \times 72 \times 1)$ cu. in.
$= 6768$ cu. in.
" " ends $= 2 \times (40 \times 47 \times 1)$ cu. in.
$= 3760$ cu. in.;
$\therefore$ volume of iron $= 13552$ cu. in.

20. Number of cubic feet in the cistern $= \dfrac{93.75 \times 112 \times 16}{1000}$.

23. The cubic content of a brick in the second case $= (\frac{4}{5})^3$ of that of a brick in the first case.

Hence, we may leave the exact dimensions of the first brick out of account and find the cost thus:—

$$\text{cost of 1 brick} = \$\left(\tfrac{64}{125} \text{ of } \frac{213.50}{9760}\right);$$

$$\therefore \text{cost of 100 bricks} = \$\left(100 \times \tfrac{64}{125} \times \frac{213.50}{9760}\right) = \$1.12.$$

25. Amount of water drawn from canal

$$= (118 \times 660 \times 48 \times \tfrac{1}{12}) \text{ cu. ft.}$$

Amount of water in the lock $= (80 \times 12 \times 8\frac{1}{2})$ " "

$$\therefore \text{number of barges} = \frac{236 \times 330 \times 4}{80 \times 6 \times 17} = 38.$$

26. Area painted for $56\frac{1}{4}s. = 50$ sq. ft.;

$\therefore$ " " " $9s. = \dfrac{9 \times 50}{56\frac{1}{4}}$ sq. ft. $= 8$ sq ft.;

$$\therefore \text{additional height} = \frac{8}{4 \times 18} \text{ ft.} = \tfrac{1}{9} \text{ ft.}$$

31. $(2 \times \text{breadth}) \times \text{breadth} \times \dfrac{\text{breadth}}{2} = 4096$ cu. ft.;

$\therefore$ cube of breadth $= 4096$ " "

35. Since the capacities of the cisterns vary as the cubes of the measures of their like dimensions;

$\therefore 1 : 3 :: 6^3 : (\text{measure of length})^3;$

$\therefore (\text{measure of length})^3 = 3 \times 6^3;$

$\therefore \text{measure of length} = 6 \times \sqrt[3]{3} = 8.653 \ldots$

37. By drawing the figure, it will be seen that

One segment of base $= \sqrt{218^2 - 120^2} = 182;$

And the other " " " $= \sqrt{241^2 - 120^2} = 209.$

38. If H represents the measure of the hypothenuse, then $H^2 = (H - 625)^2 + 3925^2;$

$$\therefore 1250 \times H = 3925^2 + 625^2.$$

41. If S is the measure of one of the equal sides of the triangle, the measure of the area of triangle $= S^2 - 13$,

and " " " " $= (S+1)^2 - 24$;

$\therefore (S+1)^2 - 24 = S^2 - 13$;

$\therefore 2S + 1 = 11$;

$\therefore S = 5$.

44. Area of circumscribed square $= (d\ )$ sq. ft.

" " inscribed " $= \{\sqrt{\tfrac{1}{2}d^2}\}^2$ sq. ft.

$= \tfrac{1}{2}d^2$ sq. ft.;

$\therefore d^2 - \tfrac{1}{2}d^2 = 338$;

$\therefore \qquad d^2 = 676$.

46. Entire length of the road $= \dfrac{4.3 \times 4840}{22}$ yd. $= 946$ yd.;

$\therefore$ perimeter of field $= (946 - 88)$ yd. $= 858$ yd.;

$\therefore$ length $\times$ (429 − length) $= 4840 \times 9$;

$\therefore$ length $= 264$ yd.,

and breadth $= 165$ yd.

60. Number of 4 in. balls $= 6^3 = 216$.

" " 6 in. " $= 4^3 = 64$.

" " 8 in. " $= 3^3 = 27$.

Cubic content of vessel $= (24^3)$ c. in. $= 13824$ c. in.

" " " 1 ball $= (\frac{22}{7} \times \frac{4^3}{6})$ c. in.

" " " 216 balls $= (216 \times \frac{22}{7} \times \frac{4^3}{6})$ c. in.

$= \frac{50688}{7}$ c. in.

Space unoccupied $= (13824 - \frac{50688}{7})$ c. in.

62. If both globes were of the same material as the first, their weights would be 16 Kg. and $\frac{5}{4}$ of 25 Kg., or $31\frac{1}{4}$ Kg.

Since the volumes of similar solids are to one another as the cubes of the measures of their corresponding lines of measurement,

$\therefore$ (measure of diameter of first)^3 : (measure of diameter of second)^3 :: 16 : $31\frac{1}{4}$;

$$\therefore \frac{(\text{measure of diameter of first})^3}{(\text{measure of diameter of second})^3} = \frac{16}{31\frac{1}{4}} = \frac{64}{125};$$

$\therefore$ measure of diameter of first : measure of diameter of second :: 4 : 5.

64. Distance in which small wheel makes 1 rev. $= (4 \times \frac{22}{7})$ ft.
" " " " " 7 " $= (28 \times \frac{22}{7})$ "
" " large " " 1 " $= (7 \times \frac{22}{7})$ "
" " " " " 4 " $= (28 \times \frac{22}{7})$ ";

$\therefore$ distance in which one wheel turns 3 times more than other $= (28 \times \frac{22}{7})$ ft.

67. Solid content will be $(\frac{5}{6})^3$ of what it was.
Hence, the diminution is $\frac{91}{216}$ " " "

68. Measure of side $\times \sqrt{3} = 8$;

$$\therefore \text{ measure of side} = \frac{8}{\sqrt{3}} = \tfrac{8}{3}\sqrt{3} = 4.6186\ldots$$

69. Representing the solid content of the cone by 1 and the part above the plane of section by $\frac{1}{2}$,

$$1 : \tfrac{1}{2} :: 20^3 : (\text{measure of height})^3;$$
$$\therefore (\text{measure of height})^3 = 20^3 \times \tfrac{1}{2};$$
$$\therefore \text{ measure of height} = \sqrt[3]{(20^3 \times \tfrac{1}{2})}$$
$$= \tfrac{20}{2}\sqrt[3]{4} = 15.874;$$

$\therefore$ the cone must be cut 15.874 in. from the apex,
or $(20 - 15.874)$ in. from the base.

70. As the part occupied by the axle is $\frac{1}{10}$ of the diameter, hence, this surface is $\frac{1}{100}$ of the circular surface of the stone, and the part to be ground off by each is $\frac{99}{400}$ of the circular surface.

After the first grinds off his part, there will remain $\frac{301}{400}$ of the surface;

$$\therefore 1 : \tfrac{301}{400} :: 60^2 : (\text{diameter remaining})^2;$$
$$\therefore (\text{diameter remaining})^2 = 60^2 \times \tfrac{301}{400},$$
$$\text{and diameter remaining} = (\tfrac{60}{20}\sqrt{301}) \text{ in.}$$
$$\text{Part ground off by the first} = 60 - 52.048\ldots \text{ in.}$$
$$= 7.951 \text{ in.}$$

MISCELLANEOUS PROBLEMS. PAGE 311

3. *A* works alone for 1 da., then *A* and *B* work together for 2 da. They will have $(\frac{1}{7} + \frac{2}{7} + \frac{2}{6})$ of work done when *C* joins, leaving $\frac{5}{21}$ of work still to do.

4. 120 m. of silk at 5.45 francs cost 654 francs, or £$\frac{3270}{127}$.

$$120 \text{ m.} = \frac{120}{.914383} \text{ yd.}$$

72 yd. at 6*s*. 7¾*d*. sell for £23 18*s*. 6*d*.
$\frac{54161644}{914383}$ yd. at 6*s*. 5*d*. sell for £19 0*s*. 1*d*.
Total selling price is £42 18*s*. 7*d*. or £42$\frac{223}{240}$.
Gain is £42$\frac{223}{240}$ − £$\frac{3270}{127}$ = £$\frac{523681}{30480}$.

10. Let cost price be 100%.
Then, if article had cost 20% less, it would have cost 80%.
Then $\frac{\text{gain}+20}{80} = \frac{\text{gain}+30}{100}$, since he would gain 20 more if it had cost only 80.

13. *A* owns .649 of the estate. *B* owns .351 of it.
$\frac{7}{15}$ of .351 of estate is worth .649 of estate − $876.
Hence, .4852 of estate is worth $876.

14. *A* can do the work in 9⅖ da.
B can do same work in $\frac{3\frac{1}{3}}{4\frac{1}{2}}$ of 9⅖ da., or $\frac{188}{27}$ da.

20. Proportion is ⅓ : $\frac{2}{7}$: ⅜, or $\frac{56}{168}$: $\frac{48}{168}$: $\frac{63}{168}$, or 56 : 48 : 63.
First gets $\frac{56}{167}$ of $1686.70.

21. Distance travelled in 1 hr. = 36 × 5280 ft.
Number of revolutions of wheel = 60 × 144.

$$\text{Circumference of wheel} = \frac{36 \times 5280}{60 \times 144} \text{ ft.}$$

22. Cost price = $1496.50.

$$\text{Selling price} = \$\frac{1496.50 \times 9}{8}.$$

A note at 90 da. for this amount is discounted at 7%.
Discount on $100 = $$(\frac{100}{1} \times \frac{7}{100} \times \frac{93}{365}) = \$\frac{651}{365}$;

$$\therefore \text{proceeds} = \$\left(\frac{1496.50 \times 9}{8} \times \frac{35849}{36500}\right).$$

24. 90 m. at $2\frac{1}{2}$ francs cost 225 francs.
Duty at 10% ad valorem is 22.5 "
∴ cost in New Orleans is 247.5 "

$$\text{or } \$\left(\frac{247.5}{1}\times\frac{1}{5.20}\right).$$

90 m. = 99 yd. (See note, page 74 of text.)

25. Value of house = $665.

In case of loss owner recovers $\frac{4}{7}$ of $\$\frac{665}{1}$, or $380.

He also recovers $\frac{5}{6}$ of premium, or $\frac{5}{6}$ of 6%, or 5%;

∴ an insurance of $100 yields $5 on account of premium and $95 on account of house.

$95 on account of house is secured by policy of $100,
$380 " " " " " " " $400.

29.
$$\frac{5}{7}=\frac{5\times 4}{7\times 4}=\frac{20}{28};$$
$$\therefore \tfrac{1}{4} \text{ of } \tfrac{5}{7}=\tfrac{1}{4} \text{ of } \tfrac{20}{28}=\tfrac{5}{28};$$
$$\therefore \tfrac{3}{4} \text{ of } \tfrac{5}{7}=\tfrac{5}{28}+\tfrac{5}{28}+\tfrac{5}{28}=\tfrac{15}{28}.$$

30. Sides are as 4 : 3; ∴ whatever multiple of 4 the greater side is, the shorter side is the same multiple of 3;

∴ the rectangle contains 12 equal squares.
12 squares = 34992 sq. ft.
1 square = 2916 " "
Side of square = 54 ft.; sides of rect. = 216 ft. and 162 ft.

31. Amount = $\$\{2400\,(1.05)^3\}$.

33. 80% of marked price = 110% of cost price.

35. Surface of sphere $= 4\pi r^2 = 1711\frac{1}{9}$ sq. in., whence $r=\frac{35}{3}$ in.
Surface of cube = area of 6 faces = 1734 sq. in.,
whence side of cube = 17 in.

38. Selling price = $\frac{6}{5}$ cost price = $600; cost price = $500.
" " = $\frac{4}{5}$ " " = $600; " " = $750.
Loss on these two = $50; ∴ total gain on third = $125;
∴ third cost $625.

39. $\frac{3+2}{102}$ of value of flour = $500.

40. Half the sum of the sides $= 60$.
Area $= \sqrt{60 \times 35 \times 21 \times 4}$.

41. Insurance cost $\$\frac{3600}{1} \times \frac{3}{4} \times \frac{8}{1000}$ or \$21.60.
Taxes are $\$\frac{4500}{1} \times \frac{2}{3} \times \frac{18}{1000}$ or \$54.00;
∴ rent per annum $= \$21.60 + \$40 + \$54 + \$225 = \$340.60$.

42. A draft on Chicago for $\$100\frac{3}{8}$ would sell for \$100 in Denver.

43. Selling price of first horse $= 120\%$ of cost price.
" " of second horse $= \frac{4}{5} \times 120\%$ of cost price
$= 96\%$ of cost price;
∴ loss $= 4\%$ of cost price of first horse.

45. Stock pays \$5 on an investment of \$100; ∴ pays \$6 on an investment of \$120.

46. Face of note $= \$\frac{730}{1} \times \frac{6}{100} \times \frac{93}{365} + \$730$.
Note is due Sept. 9th. Discounted July 2nd.
Discount $= \$\frac{73930}{100} \times \frac{6}{100} \times \frac{69}{365}$.

47. $\frac{1}{3}$ *A*'s money $= \frac{1}{4}$ *B*'s money $+ \$10$.
A's " $= \frac{3}{4}$ *B*'s " $+ \$30$.
A's and *B*'s " $= \frac{7}{4}$ *B*'s " $+ \$30 = \$310$.

50. 10 ac. $= 1600$ sq. rd.; ∴ side of square field $= 40$ rd.
$\pi r^2 = 1600$ sq. rd., whence r may be found.
Circumference $= 2\pi r$.

51. Diagonal of square will be diameter of circle, and the two diagonals intersect at right angles.

52. Loss $= \frac{1}{10}$ of proceeds $= \frac{1}{11}$ of cost $= \$1$.
Gain $= \frac{11}{21}$ " " $= \frac{11}{32}$ " " $= \$11$.
Total gain $= \$10$ on a total cost of \$43.

53. To cross the bridge the train must travel the length of the bridge + its own length.

55. Number of cu. m. = 231 × 157 × .000016386.
" " litres = 231 × 157 × .000016386 × 1000.

56. Number of bushels purchased = 59.50 ÷ .70.
" " " sold = 78.40 ÷ 1.12.

59. He sells one share of \$100 at 89. Income is \$4.
Invests amount in stock at 102. Income is $\$(\frac{5}{1} \times \frac{89}{102})$.
Income is increased $\$\frac{37}{102}$ by selling 1 share.

61.
$$20 : 10 :: 4^3 : d^3;$$
$$\therefore d = 2\sqrt[3]{4}$$
$$= 3.1748\ldots$$

62. 63^3 cu. in. = 16^3 cu. dcm. = 4096 l.

63. The grocer cheats $1\frac{9}{16}$ oz. in 16 oz.;

$\therefore$ he cheats $\frac{1\frac{9}{16}}{16}$ of \$73.92.

64. *A* and *B* do $\frac{1}{8}$ of work in 1 da.
B " *C* " $\frac{1}{9}$ " " " 1 "
C " *A* " $\frac{1}{7}$ " " " 1 "
$\therefore$ 2 *A*'s, 2 *B*'s and 2 *C*'s do $(\frac{1}{8} + \frac{1}{9} + \frac{1}{7})$, or $\frac{191}{504}$ in 1 da.;
$\therefore$ *A* does $\frac{1}{2}$ of $\frac{191}{504} - \frac{1}{9}$, or $\frac{79}{1008}$ in 1 da.

65. Area of rectangle of which the base and perpendicular are adjacent sides is (30 × 40) sq. ft.

The area of rectangle of which the hypothenuse and perpendicular upon it are the dimensions is the same;

hence, measure of perpendicular $= \frac{30 \times 40}{50} = 24$;

$\therefore$ perpendicular = 24 ft.

68. 6 premiums at $1\frac{1}{4}\% = 7\frac{1}{2}\%$.
$7\frac{1}{2}\%$ of $\frac{4}{5}$ of value of house $= \frac{3}{50}$ of value;
$\therefore$ total loss $= (\frac{1}{5} + \frac{3}{50})$ of value of house = \$1945.

69. Interest on \$314 at $1\frac{1}{2}\%$ for 1 yr. = \$4.71;
$\therefore$ interest on (\$460 + \$314) at certain rate is (\$43.41 − \$4.71).

71. He gives $15\frac{5}{8}$ oz. for 1 lb.;

$\therefore$ he gains $\frac{\frac{3}{8}}{15\frac{5}{8}}$ of \$62.50.

72. Area = area of surface of cylinder + twice area of surface of hemisphere

$$= \left(\frac{44}{7} \times \frac{5}{2} \times \frac{6}{1} + \frac{4 \times 22}{7} \times \frac{25}{16}\right) \text{ sq. ft.}$$

73. $(\frac{4}{3} \times \frac{22}{7} \times r^3)$ cu. ft. = 8 cu. ft.,

whence radius of sphere = $\sqrt[3]{(\frac{21}{11})}$ = etc.

75. $\frac{3}{4}\%$ of $\frac{2}{3}$ of value of house = \$40.50.

76. 1 in. = 2.539954 cm.

1 cu. in = $(2.539954)^3$ cu. cm. and weighs $\frac{1000}{1728}$ oz.;

$\therefore$ 1 cu. cm. weighs $\left(\frac{1000}{1728} \times \frac{1}{(2.539954)^3}\right)$ oz.

$$= \left(\frac{1000}{1728} \times \frac{1}{(2.539954)^3} \times \frac{28}{1}\right) \text{ g.}$$

81. *A*'s share = $\frac{5}{7}$ of *B*'s share + \$17;

$\therefore$ *A*'s share + *B*'s " = $\frac{12}{7}$ of *B*'s share + \$17 = \$269;

$\therefore$ $\frac{12}{7}$ of *B*'s " = \$252.

82. $\frac{4}{5}$ of selling price = 93% of cost price;

$\therefore$ " " = $\frac{5}{4}$ of 93% of cost price.

83. Selling price = 90% of cost price;

$\therefore$ 90% of cost price + \$7.50 = 110% " "

$\therefore$ 20% of cost price = \$7.50.

84. \$6 income is obtained by investing \$$91\frac{1}{2}$.

\$304 " " " " " \$$(\frac{183}{2} \times \frac{304}{6})$.

Income obtained by investing \$76 is \$$4\frac{1}{2}$,

" " " " \$$(\frac{183}{2} \times \frac{304}{6})$ is \$$(\frac{9}{2} \times \frac{1}{76} \times \frac{183}{2} \times \frac{304}{6})$.

85. Net income = \$1728.40; amount exempt = \$400;

$\therefore$ $\frac{984}{1000}$ of taxable part of income = \$1328.40.

86. Volume of water to raise level in reservoir 2 in.

$= (156 \times 120 \times \frac{2}{36})$ cu. yd.

Area of cross section of pipe $= (\frac{22}{7} \times \frac{7}{1} \times \frac{7}{1})$ sq. in.

$= \frac{154}{144 \times 9}$ sq. yd;

$\therefore$ length of pipe filled with water $= \left(156 \times 120 \times \frac{2}{36} \div \frac{154}{144 \times 9}\right)$ yd;

$\therefore$ time required $= \left(\frac{156 \times 120 \times 2}{36} \times \frac{144 \times 9}{154} \times \frac{1}{8 \times 1760}\right)$ hr.

87. $\frac{4 + 2\frac{1}{4}}{102\frac{1}{4}}$ of sales $=$ \$350.

88. Volume $= (240 \times 300 \times 75)$ cu. cm.

1 cu. cm. of water $= 1$ gram.;

$\therefore$ 1 " " " alcohol $= \frac{4}{5}$ "

$\therefore$ number of kilograms of alcohol $= 240 \times 300 \times 75 \times \frac{4}{5} \times \frac{1}{1000}$.

89. 24.25 francs $=$ £1 $=$ \$$\frac{40}{9} \times \frac{109\frac{1}{2}}{100}$.

90. Area of triangle $= \sqrt{70 \times 35 \times 28 \times 7}$ sq. ft.,

$= (490\sqrt{2} \times 9.2899683)$ sq. dm.;

$\therefore$ volume of water $= (490\sqrt{2} \times 9.2899683 \times \frac{1}{10})$ cu. dm.,

and 1 cu. dm $=$ 1 l.

92. (25×120) sq. ft. sell for \$$(15 \times 25)$.

93. Rate of train per hour $= (\frac{10800}{25} \times \frac{60}{1})$ m.

$= \left(\frac{10800}{25} \times \frac{60}{1} \times \frac{1}{.914383}\right)$ yd.

95. 1 man would do whole work in $(9 \times 24 \times \frac{4}{3} \times 20)$ hr.;

$\therefore$ 12 men would do $\frac{2}{3}$ work in $(9 \times 24 \times \frac{4}{3} \times 20 \times \frac{2}{3} \times \frac{1}{12} \times \frac{1}{10})$ da.

96. *C* runs 95 yd. while *B* runs 100 yd.;

$\therefore$ *C* " $180\frac{1}{2}$ " " *B* " $(\frac{100}{95} \times \frac{36}{2})$ ft., or 190 yd.;

$\therefore$ *A* will run 200 yd. while *B* runs 190 yd.

99. $\$90$ yield income of $\$4$.
100 " " " $4\frac{4}{9}$.
110 " " " 5.
100 " " " $4\frac{6}{11}$;
∴ 5% stock at 110 is better investment.

102. See hint to question number 64.

103. $\text{Income} = \$\left(6\frac{1}{2} \times \frac{12450}{1} \times \frac{1}{194\frac{1}{2}}\right) + \$\left(6\frac{1}{4} \times \frac{12850}{1} \times \frac{1}{198\frac{1}{2}}\right)$.

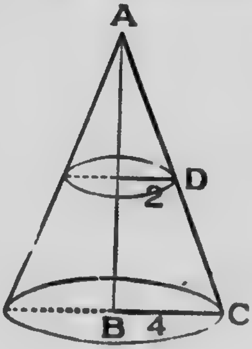
104. Let *A*'s investment be $4 and *B*'s $5,
then *A* has $4 for 6 mo. + $\frac{3}{5}$ of $4 for 6 mo. = $$38\frac{2}{5}$ for 1 mo.
" *B* " $5 " 6 " + $\frac{2}{5}$ of $5 " 6 " = $42 " 1 "

$$\therefore A\text{'s share of gain is } \frac{38\frac{2}{5}}{38\frac{2}{5}+42} \text{ of } \$4050.$$

108. Volume = (20 × 20 × 4) cu. dm.;
∴ vessel will contain 1600 l. of water.

109. Selling price = $87\frac{1}{2}\%$ of $4720.
Amount realized = $\frac{85}{100}$ of $\frac{7}{8}$ of $4720,
and cost price = $4720.

110. Since the straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to half of it;



∴ the radius of the upper end of the frustum will be 2 in.

$$\text{Volume} = \left\{\left(\frac{22}{7} \times 2^2 + \frac{22}{7} \times 4^2 + \sqrt{\frac{22}{7} \times 2^2 \times \frac{22}{7} \times 4^2}\right) \times \frac{5}{1} \times \frac{1}{3}\right\} \text{ cu. in.}$$

Since AB = 10 and BC = 4,

$$\therefore AC = \sqrt{116}; \quad \therefore CD = \tfrac{1}{2}\sqrt{116}.$$

$$\text{Then area} = \left(\frac{22}{7} \times \frac{4}{1} + \frac{22}{7} \times \frac{8}{1}\right) \times \frac{1}{2} \times \frac{1}{2}\sqrt{116} + \frac{22}{7} \times 2^2 + \frac{22}{7} \times 4^2) \text{ sq. in.}$$

111. Cost price of stock = $82\frac{1}{2}$; selling price = $81\frac{1}{2}$.
Loss = $1 per share of $100.
Half-yearly income is $3;
∴ $2 is net increase of capital from 1 share of $100.
$250 " " " " 125 shares of $100.
Amount invested = $$(82\frac{1}{2} \times 125)$.

112. 10 boys and 6 girls finish the work in 2 da.

$$10 \text{ " " } 6 \text{ " do } \left(\frac{10}{12 \times 25} + \frac{6}{18 \times 25}\right) \times 2,$$

or $\frac{7}{75}$ of work in 2 da.;
∴ all working together do $\frac{68}{75}$ of the work; and do $\frac{3}{25}$ in 1 da.;
∴ time = $(\frac{68}{75} \div \frac{3}{25}) + 2$, or $9\frac{5}{9}$ da.

113. Interest on mortgage = $90.
Insurance premium = $33.75.
Taxes = $72;
∴ net income from rent = $344.25;
∴ net income from salary = $1788.80;
∴ net income from taxable part of salary = $1388.80,
$\frac{992}{1000}$ of taxable part of salary = $1388.80.

114. If whole sum were loaned at 6% the interest would be $60;
∴ $\frac{1}{2}$% less on one portion realizes $2.10 less interest;
∴ $\frac{1}{2}$% of sum loaned at $5\frac{1}{2}$% = $2.10;
∴ sum loaned at $5\frac{1}{2}$% = $420.

115. 2% on $\frac{2}{3}$ of the risk = $1\frac{1}{3}$% on the whole risk;
∴ premium received exceeded premium paid by $1\frac{1}{20}$%;
$1\frac{1}{20}$% of the risk = $42.

116. Length of shadow : length of pole :: 4 : 3;
∴ a pole $48\frac{1}{2}$ ft. long casts a shadow $(48\frac{1}{2} \times \frac{4}{3})$ ft. long.

117. Income from $4000 of $3\frac{1}{2}$% stock = $140;
∴ " " 4% stock = $150;
∴ number of shares of 4% stock = $\frac{150}{4}$.
$\frac{150}{4}$ shares cost $$\{(80\frac{1}{4} - \frac{1}{4}) \times 4\}$, or $3200,
1 share costs $$(\frac{3200}{1} \times \frac{4}{150})$, or $85\frac{1}{3}$;
∴ market value of 4% stock = $$(85\frac{1}{3} - \frac{1}{4})$.

118. 3% of commission = $10.80, whence commission = $360.

$\frac{3+3}{103}$ of amount of sales = $360.

$\frac{97}{103}$ " " " " $= \$\left(\frac{360}{6} \times \frac{97}{1}\right)$ = amount invested.

120. The square on the diagonal = twice area of the courtyard.

121. See solution of number 114.

122. $22\frac{2}{3}\%$ of value of unbroken bottles = $499.20.

125. One side of the square field = $\sqrt{77841}$ yd.;

$\therefore$ length of walk = $(279 \times 4 + 4 \times 4)$ yd.

126. The water flows 12 mi. per hour;

$\therefore$ 6 mi. of pipe will be emptied in half an hour;

$\therefore$ volume of water emptied into reservoir

$$= \left(\frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{1}{144} \times \frac{1}{9} \times \frac{6 \times 1760}{1}\right) \text{cu. yd.};$$

$\therefore$ surface of reservoir is raised

$$\left\{\frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{1}{144} \times \frac{1}{9} \times \frac{6 \times 1760}{1} \div (210 \times 56)\right\} \text{yd.}$$

127. 15 gal. at 52c. = $7.80.
13 " at 40c. = $5.20;
$\therefore$ he sells mixture for $18.

At 60c. a gallon he must sell 30 gal. He adds 2 gal. of water.

128. The first 7 questions receive 25 marks each;

$\therefore$ 75 marks are to be divided among the last three;

$\therefore$ question number 8 receives $\frac{3}{7+5+3}$ (or $\frac{3}{15}$) of 75 marks.

132. $\frac{76}{100}$ of longer part = $\frac{19}{15}$ of shorter, whence longer = $\frac{5}{3}$ of shorter.

Hence, longer + shorter = $\frac{8}{3}$ shorter = 120 ft..

134. Interest on $350 for $5\frac{1}{2}$ yr. = $115.50.

" " $100 " 1 " $= \$\left(\frac{115.50}{1} \times \frac{2}{11} \times \frac{100}{350}\right) = \$6$;

$\therefore$ rate is 6%.

136. $2\frac{3}{5}\%$ of $\frac{3}{4}$ of value of wheat = \$88.14.
Value of wheat = $\$(88.14 \times \frac{500}{13} \times \frac{4}{3})$.

137. 10000 lb. pork at $8\frac{1}{2}$c. = \$850.
Commission + freight = $3\frac{1}{2}\%$ of \$850 + \$19.55 = \$49.30;
∴ amount of lumber purchased = $(800.70 \times \frac{100}{102} \times \frac{1}{20})$ thousand ft.

140. See table of linear measurement, page 74.

142. Volume of ice = $(\frac{4840}{2} \times \frac{9}{1} \times \frac{3}{12})$ cu. ft.;
∴ volume of water = $(\frac{4840}{2} \times \frac{9}{1} \times \frac{3}{12} \times \frac{10}{11})$ cu. ft.

145. See section 111, page 298.

146. This is simply finding the hypothenuse of a right-angled triangle whose sides containing the right angle are 7 ft. and 24 ft.

147. Interest on \$150 for 3 mo. = interest on \$450 for 1 mo.
= " " \$90 " 5 mo.

148. Amount received from bank = \$3000 less 5% of \$3000.
Interest on \$2850 for 1 yr. at $5\frac{1}{2}\%$ = \$156.75.

149. 60 shares at \$97 cost \$5820.
42 " " $\$102\frac{1}{2}$ realize \$4305.
18 " " $\$99\frac{1}{3}$ " \$1788.

151. Capital at end of first year = 140% of \$4000.
" " " " second " = $85\frac{5}{7}\%$ of \$5600.
" " " " third " = $133\frac{1}{3}\%$ of \$4800;
∴ average annual gain = $\$\frac{2400}{3}$, or \$800.

152. There are 13500 oz. of mercury in 1 cu. ft.
" " 2000 × 16 oz. " " $\frac{2000 \times 16}{13500}$ cu. ft;
∴ number of gallons = $\frac{2000 \times 16}{13500} \times \frac{1728}{1} \times \frac{1}{277.274}$.

153. Original volume = 1^3 or 1.
Increased " = $(\frac{13}{12})^3$ or $\frac{2197}{1728}$ since $8\frac{1}{3}\% = \frac{1}{12}$;
∴ increase in " = $\frac{469}{1728}$ or 27.14%.

154. Note is due June 23rd.

Number of days from March 20 to June 23 = 95.

$(1 × $\frac{6}{100}$ × $\frac{95}{365}$) is discount on $1.

$11.40 is discount on $$\left(1 \times \frac{100}{6} \times \frac{365}{95} \times \frac{11.40}{1}\right)$.

157. See example 71, page 18.

158. 6000 francs = £$\frac{6000}{25.60}$ = $$\left(\frac{40}{9} \times \frac{219}{200} \times \frac{6000}{25.60}\right)$.

159. *B* gets 10% on $3000 and *A* gets a salary of $1000.
A gets ½ of balance of profits = ½ of $(7460 − 1300);
∴ in all *A* gets $1000 + $3080;
∴ he may withdraw $4080 − $750 = $3330.

160. In 3⅓ min. A, B and C together fill $(\frac{1}{24} + \frac{1}{10} + \frac{1}{27}) \times 3\frac{1}{3}$, or $\frac{193}{324}$;
∴ B must fill $\frac{131}{324}$;
∴ time required is $\frac{131}{324}$ of 10 min.

161. 427 florins = (2.6 × 427) francs

$$= \frac{2.6 \times 427}{4} \text{ roubles.}$$

162. 8 hr. 59 min. 52 sec. $= 8 \times 15° + 59 \times \frac{1}{4}° + 52 \times \frac{1}{4}'$
$= 134° \ 58'$;
∴ Rome is 134° 58′ east of San Francisco;
∴ longitude of Rome is 12° 27′ east of prime meridian.

165. Let T = time, R = rate, I = interest, P = principal.

Then $\frac{T \times R}{100}$ = interest when P = $1,

I = " " P = $$\frac{1 \times 100 \times I}{T \times R}$.

166. In 1⅓ min. the hare runs 352 yd.;
∴ the hound has to gain (352 + 40) yd.
He gains $\frac{1760 \times 3}{2}$ yd. in 60 min.
" " 392 " " $8\frac{10}{11}$ " ;
∴ the hare runs $\left(\frac{9 \times 1760}{60} \times \frac{98}{11} + 352\right)$ yd.

167. Discount on \$1 for 63 dy. at 6% = \$1 × $\frac{6}{100}$ × $\frac{63}{365}$ = \$$\frac{378}{36500}$;
∴ \$$\frac{36122}{36500}$ is proceeds when face of note is \$1.

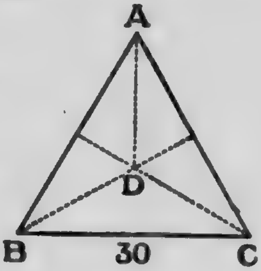
168. An increase in selling price of 35c. per bushel increases receipts by \$287,

Or, the whole quantity at 35c. a bushel = \$287;
∴ number of bushels = $\frac{28700}{35}$.

169. $\frac{4}{5}$ of *A*'s vote = $\frac{5}{6}$ of *B*'s vote.
A's " = $\frac{25}{24}$ " *B*'s "
∴ *A*'s vote + *B*'s " = $\frac{49}{24}$ " *B*'s " = 735;
∴ *A* gets $\frac{25}{49}$ of total vote and *B* gets $\frac{24}{49}$.

172. By drawing the figure it will be seen that the field consists of a rectangle and a right-angled triangle.

173. Triangles ADC and BDC may be proven equal (Euc. 1, 4); also triangles ADB and BDC;
∴ triangle BDC is $\frac{1}{3}$ of the triangle ABC.



174. \$$(91\frac{1}{2}+\frac{1}{8})$ buys 1 share of \$100.
\$8063 " 88 shares " \$100.
Net income = \$$3\frac{1}{2} \times 88 \times \frac{995}{1000}$.

175. 1 man digs $(33\frac{3}{4} \times 2\frac{2}{3} \times 2\frac{5}{8})$ cu. yd. in $(94\frac{1}{2} \times 10 \times 12)$ hr.
1 " " $(23\frac{1}{4} \times 2\frac{1}{3} \times 1\frac{5}{6})$ " " "

$$\left(94\frac{1}{2} \times 10 \times 12 \times \frac{23\frac{1}{4} \times 2\frac{1}{3} \times 1\frac{5}{6}}{33\frac{3}{4} \times 2\frac{2}{3} \times 2\frac{5}{8}}\right) \text{ hr.}$$

217 men dig $(23\frac{1}{4} \times 2\frac{1}{3} \times 1\frac{5}{6})$ cu. yd. in 11 da. of

$$\left(94\frac{1}{2} \times 10 \times 12 \times \frac{23\frac{1}{4} \times 2\frac{1}{3} \times 1\frac{5}{6}}{33\frac{3}{4} \times 2\frac{2}{3} \times 2\frac{5}{8}} \times \frac{1}{217 \times 11}\right) \text{ hr. each.}$$

176. Sum received for the goods = $\frac{90}{100}$ of cost.
$\frac{90}{100}$ of cost + \$600 = $\frac{102}{100}$ " "
∴ cost = \$5000.
Hence $\frac{125}{100}$ of original cost = \$5000,
and " " = \$4000.

179. Semi-annual income from Bank of Montreal stock

$$= \$500.$$

" " " Dominion Bank stock

$$= \$\frac{185 \times 100}{108} \times \frac{7}{3}.$$

180. When number is increased 20% it becomes 120% of number.

This amount increased $16\frac{2}{3}\%$ becomes $116\frac{2}{3}\%$ of 120% of number

$$= 280;$$

$\therefore$ 140% of number = 280.

181. Let dimensions be $4x$, $3x$ and $2x$ ft.

Then $24x^3 = 3000$.

Whence $x = 5$.

182. 9% of capital = \$22500.

10% " " = \$25000;

$\therefore$ incomes are $\frac{4}{20}$ of \$25000, etc.

183. He sells $\frac{1}{3}$ of \$750 worth for 96% of \$250, or \$240;

$\therefore$ he must sell \$500 " " \$(780 − 240), or \$540.

At old selling price he would sell \$500 for 96% of \$500, or \$480;

$\therefore$ he increases selling price \$60 in \$480, or $12\frac{1}{2}\%$.

184. 1 man digs $(72 \times 18 \times 12)$ cu. yd. in $(8 \times 16 \times 36)$ hr.

1 man digs $(64 \times 27 \times 18)$ cu. yd. in $\left(8 \times 16 \times 36 \times \frac{64 \times 27 \times 18}{72 \times 18 \times 12}\right)$ hr.;

$\therefore$ number of men required to dig it in 24 da. of 8 hr. each

$$= 8 \times 16 \times 36 \times \frac{64 \times 27 \times 18}{72 \times 18 \times 12} \times \frac{1}{8 \times 24}.$$

185. See page 72 of text.

186. Difference in longitude = 193° 32′.

" " time $= 193 \times 4$ min. $+ 32 \times 4$ sec.

= 12 hr. 54 min. 8 sec.

Washington time is slower than Pekin time.

187. See hint on solution of number 69.

190. 6% for 4 yr. and 8 mo. = 28% for 1 yr.

28% of money = \$497. Total amount = \$1775.

$\frac{2}{3}$ A's = $\frac{3}{4}$ B's;

∴ A's = $\frac{9}{8}$ B's.

$\frac{1}{2}$ B's = $\frac{3}{5}$ C's;

∴ B's = $\frac{6}{5}$ C's;

∴ A's = $\frac{9}{8}$ of $\frac{6}{5}$, or $\frac{27}{20}$ C's;

∴ A's + B's + C's shares = $(\frac{27}{20} + \frac{6}{5} + 1)$ times C's share.

$\frac{71}{20}$ times C's share = \$1775.

191. House is insured for $\frac{4}{7}$ of \$665, or \$380.

Owner wishes to recover $\frac{5}{6}$ of premium. $\frac{5}{6}$ of 6% = 5%;

∴ \$95 of value of house is secured by \$100 of insurance.

192. A invests \$5 for 4 mo. + $\frac{1}{2}$ of \$5 for 8 mo. = \$40 for 1 mo.

B " \$8 " 4 " + $\frac{1}{3}$ " \$8 " 8 " = \$53$\frac{1}{3}$ " 1 "

$$\therefore A \text{ gets } \frac{40}{40 + 53\frac{1}{3}} \text{ of } \$4000.$$

193. Amount of sales = \$(6.37$\frac{1}{2}$ × 800).

$$\text{Amount of commission} = \$\left(6.37\tfrac{1}{2} \times 800 \times \frac{3+2}{102}\right).$$

194. Premium paid first company = $\frac{1}{300}$ of $\frac{1}{5}$ of $\frac{3}{4}$ of \$10000.

" " second " = $\frac{1}{500}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of \$10000.

" " third " = $\frac{3}{700}$ of $\frac{7}{15}$ of $\frac{3}{4}$ of \$10000.

196. Area of hexagonal base = sum of areas of 6 equilateral triangles

$= 6\sqrt{1.2 \times .4 \times .4 \times .4}$ sq. m.

= 1.66272 sq. m.;

∴ volume of water required = 3.32544 cu. m. = 3325440 cu. cm.

= 3325440 grams = 3325.44 kg.

197.

$$\text{Number of strips} = \frac{18 \times 12}{27} = 8.$$

$$\text{Number of yards of carpet} = \frac{83}{1.50}.$$

$$\text{Number of yards of carpet per strip} = \frac{83}{8 \times 1.50} = \frac{83}{12};$$

$$\therefore \text{waste per strip} = (\tfrac{83}{12} - \tfrac{20}{3}) \text{ yd.}$$

200. After spending $\frac{3}{5}$ of money − \$10, he has left $\frac{2}{5}$ of money + \$10; ∴ $\frac{2}{5}$ of money + \$10 = $\frac{3}{10}$ of money + \$15.

201. The area of the walk is the difference in area of two circles whose radii are 90 ft. and 115 ft. respectively.

203. *B* invests \$4000 for 7 mo. + \$10000 for 5 mo., or \$78000 for 1 mo.

\$7800 is income from \$78000 invested for 1 mo.
\$5800 " " " \$58000 " " 1 "

A invested \$6000 for 5 mo. + a certain sum for 7 mo., or \$58000 for 1 mo.;

∴ a certain sum invested 7 mo. = \$28000 invested for 1 mo.;
∴ *A* had \$4000 during 7 mo. He withdrew \$2000.

204. 11520 marcs banco $= (2\frac{1}{8} \times 11520)$ francs
$= (\frac{17}{8} \times \frac{11520}{1} \times \frac{100}{518})$ dollars.

205. Selling price of $\frac{1}{3}$ of goods $= \frac{120}{100}$ of $\frac{1}{3}$ of cost, or $\frac{2}{5}$ of cost.
" " " $\frac{2}{5}$ " " $= \frac{116\frac{2}{3}}{100}$ " $\frac{2}{5}$ " " " $\frac{7}{15}$ " "
" " " $\frac{4}{15}$ " " $= \frac{90}{100}$ " $\frac{4}{15}$ " " " $\frac{6}{25}$ " "
∴ selling price $= (\frac{2}{5} + \frac{7}{15} + \frac{6}{25})$ of cost = \$1245.

206. Amount of note $= \$(\frac{219}{1} + \frac{219}{1} \times \frac{92}{365} \times \frac{5}{100}) = \$221.76$.
Proceeds $= \$221.76 - \$(221.76 \times .07 \times \frac{67}{365})$.

207. Amount of sales exceeds amount invested by amount of total commission;

∴ 5% of total commission = \$69.50.
$\frac{10}{105}$ of value of flour = \$1390.

208. Commission on \$240 = \$6.

209. See solution for number 145, page 106.

210. 1.65% of taxable income = \$33.

212. £1 $= \$\frac{3066}{630}$.
Increase over old par $= \$(\frac{3066}{630} - \frac{40}{9}) = \$\frac{266}{630}$;
∴ quotation for sterling exchange $= \frac{266}{630} \times \frac{9}{40} \times \frac{100}{1}\% = 9\frac{1}{2}\%$.

213. See Art. 87, page 265, and also solution to Exercise LIII., page 151 of Handbook.

214. 1 man does whole work in $(9 \times 15 \times 10 \times \frac{6}{5})$ hr.

$$12 \text{ men do } \tfrac{2}{3} \text{ of " " " } \frac{9 \times 15 \times 10 \times 6 \times 2}{5 \times 12 \times 8 \times 3} \text{ da.}$$

217. The square on the diagonal is twice the area of the given square.

$$\pi r^2 = \frac{42^2}{2}. \quad \text{Find } 2\pi r.$$

218. First lot cost 100% of cost and sold for 70% of cost.

Second lot cost 40% of cost of first and sold for 60% of cost of first;

∴ the two lots cost 140% of cost of first and sold for 130% of cost of first.

219. Amount of debt at end of first year $= \$15750 - \$2500$
$= \$13250$, etc.

220. $\$\frac{6 \times 48}{365}$ is discount on \$100.

221. Cost of beef $= (6\frac{3}{8} \times 252 + 17\frac{1}{2})$ c.

252 lb. at $7\frac{1}{4}$c. would cost \$18.27;

∴ number of pounds in fore-quarter $= (18.27 - 16.24) \div (7\frac{1}{4} - 5\frac{1}{2})$.

222. Total cost = cost of $(400 + \frac{1600}{6})$ lb. of tea.

$$\text{Total sales} = \text{value of } \left(\frac{400 \times 4}{3} + \frac{1600}{6} \times \frac{4}{5}\right) \text{ lb. of tea;}$$

∴ value of 80 lb. of tea = \$60.

223. Breadth of river $= \sqrt{65^2 - 50^2}$.

225.

$$3 \text{ Kg.} = 3000 \text{ g.} = \frac{3000}{8.85} \text{ cu. cm.}$$

$$\text{Length of wire} = \left(\frac{3000}{8.85} \times \frac{1}{.0314159}\right) \text{ cm.}$$

226. $\$97\frac{1}{2}$ of value of the building is covered by \$100 insurance.

$\$\frac{5850 \times 4}{5}$ " " " " " "

$$\$\left(\frac{100}{1} \times \frac{2}{195} \times \frac{5850 \times 4}{5}\right) \text{ insurance.}$$

227. Volume of reservoir $= (4 \times 6\sqrt{120 \times 40 \times 40 \times 40})$ cu. m.

Length of pipe to hold this volume $= (4 \times 6\sqrt{120 \times 40 \times 40 \times 40} + .0314159)$ m.

And this represents distance water travels in 12 hr.

230. Discount for 63 da. = \$11.34.
" " 365 " = \$65.70.

231.
$$\frac{9 + \text{number}}{17 + \text{number}} = \tfrac{2}{3}.$$
27 + 3 times the number = 34 + twice the number.

237. \$1 amounts to $\$\dfrac{404.94544}{340}$ in 3 yr.
\$1 " " $\sqrt[3]{1.1910016}$ in 1 yr.

241.
$$\frac{11 - \text{number}}{23 - \text{number}} = \tfrac{2}{5};$$
$\therefore$ 55 − 5 × number = 46 − 2 × number.

243. $83\frac{1}{3}\%$ of \$2448 worth of goods sell for $106\frac{1}{4}\%$ of \$2448 worth.
\$100 " " " " " $\dfrac{106\frac{1}{4}}{83\frac{1}{3}}$ of \$100.

246. See solution 237.

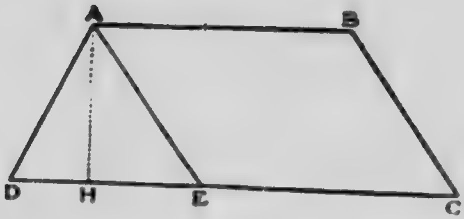
248. *A*'s share = 3 times *B*'s share; *C*'s share = $\frac{2}{3}$ of *B*'s share.

249. Let x = amount of silver added,
then $\frac{9}{20}(1 + x) = \frac{3}{10} + x$.

251. Interest on \$100 for 73 da. at 7% = \$1.40;
$\therefore$ interest on \$98.60 for 73 da. = \$1.40.
" " \$100 " 1 yr. $= \$\left(\dfrac{1.40}{1} \times \frac{365}{73} \times \dfrac{100}{98.60}\right).$

255. Surface area $= \frac{120}{100} \times \dfrac{112\frac{1}{2}}{100}$ of former area.

Since the cubic contents of the solid have not been changed,
$\therefore$ the thickness is now $\dfrac{100 \times 100}{120 \times 112\frac{1}{2}}$ of the former thickness.



266. Draw AE parallel to BC. Then AE = 15 rd. and DE = 12 rd. Hence $\sqrt{AD^2 - AH^2} + \sqrt{AE^2 - AH^2} = 12$;

$$\text{or } \sqrt{12^2 - AH^2} + \sqrt{15^2 - AH^2} = 12,$$

$$\text{and } AH = 11.709.$$

268. 2500 lb. at 6¼c. cost $156.25.

$$\text{Selling price} = \left(\frac{2500 \times 16}{15\frac{1}{2}} \times \tfrac{25}{4} \times \tfrac{120}{100}\right) \text{c.}$$

$$\text{Gain due to defective scales } \left(\frac{\frac{1}{2}}{15\frac{1}{2}} \text{ of } \tfrac{2500}{1} \times \tfrac{25}{4} \times \tfrac{120}{100}\right) \text{c.}$$

275. Suppose deposit in 1902 = $100,
then " " 1901 = $90,
and " " 1903 = $125;
also " " 1904 = $$\$\left(\tfrac{1}{3} \text{ of } (100 + 90 + 125) \times \frac{116\frac{2}{3}}{100}\right)$$
= $122.50.

The total deposits are $437.50 for every $90 deposited in 1901.

276. Length of edge = $\sqrt[3]{6^3 + 8^3 + 10^3}$ cm.

278. ($\frac{1}{3}$ of $\frac{2}{5}$ + $\frac{1}{4}$ of $\frac{3}{5}$) of cost of goods = ($\frac{1}{4}$ of $\frac{2}{5}$ + $\frac{1}{3}$ of $\frac{3}{5}$) of cost + $78.

279. Number of yards of carpet = $57.60 ÷ $1.20;

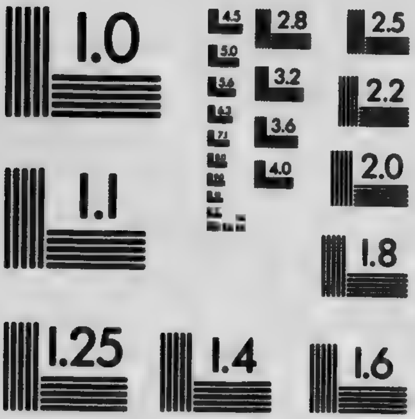
∴ area of room = $(\frac{48}{1} \times \frac{3}{1} \times \frac{21}{12})$ sq. ft.
= 252 sq. ft.

Area of room 9 ft. by 7 ft. = 63 sq. ft.
= ¼ of 252 sq. ft.;

∴ room is 18 ft. long and 14 ft. wide.

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



APPLIED IMAGE Inc
1653 East Main Street
Rochester, New York 14609 USA
(716) 482 - 0300 - Phone
(716) 288 - 5989 - Fax

282. A's gain $= \$360$ per mo., B's $\$450$ and C's $\$255$;

$\therefore A$'s capital $= \frac{360}{1065}$ of $\$\frac{35500}{1}$.

285. Interest at 6% for 63 da. $= \frac{378}{36500}$ of face of note;

$\therefore \frac{36122}{36500}$ of face of note $= \$750$.

Face of note $= \$757.85$.

$\frac{36302}{36500}$ of face of second note $= \$507.85$.

291. Each man walks $(3 \times 1760 + 44 + 249)$ yd. in $56\frac{1}{6}$ min.

293. Suppose 1 share of \$100 of stock in A is sold and invested in B.

The income will be $\$(\frac{11}{8} \times \frac{4}{129} \times \frac{405}{4})$.

Then $\$4\frac{1}{2} - \$(\frac{11}{8} \times \frac{4}{129} \times \frac{405}{4})$ is change in income resulting from transfer of 1 share of A to B.

296. Slant height of cone $= \sqrt{3 \times 1}$ ft.

$= 2$ ft.

Surface of cone $= (\frac{1}{2}$ of $\frac{44}{7} \times \frac{2}{1} + \frac{22}{7})$ sq. ft.;

$\therefore$ surface of sphere $= \frac{33}{7}$ sq. ft.

$4\,\pi r^2 = \frac{33}{7}$, whence radius of sphere may be found.

298. Deposit Dec. 31, 1890, amounts to $\$\{350\,(1.04)^4\}$
" " 31, 1891, " " $\$\{350\,(1.04)^3\}$, etc.;

$\therefore$ total amount at credit on January 1st, 1895,

$= \$[350\{(1.04)^4 + (1.04)^3 + (1.04)^2 + (1.04) + 1\}]$.

299. 10000 guilders $= £\dfrac{10000 \times 4}{45}$ in London

$$= \$\left(\frac{40}{9} \times \frac{109\frac{1}{4}}{100} \times \frac{10000 \times 4}{45}\right) \text{ in Montreal.}$$

But broker charges $1\frac{1}{4}$% for transmission from London;

$$\therefore \text{ merchant would receive } \$\left(\frac{40}{9} \times \frac{437}{400} \times \frac{10000 \times 4}{45} \times \frac{395}{400}\right).$$

300. Account sales of 1000 sheep received from *A*, October 1st, to be sold on commission.

		Sales.				
Oct.	3	250 head at $5.25			1312	50
	4	525 " " $6.00			3150	00
	4	5 " killed by accident.				
	6	220 " at $5.00..........			1100	00
		Charges.				
Oct.	1	Freight—1000 head at 17½c.	175	00		
	3	Feed and yard—250 head at 2¼c. for 2 da.	11	25		
	4	Feed and yard—530 head at 2¼c. for 3 da.	35	78		
	6	Feed and yard—220 head at 2¼c. for 5 da.	24	75		
		Commission—10% of $815.72	81	57		
		Balance remitted..........	5234	15		
			5562	50	5562	50

302. See solution of No. 192.

308. Volume of water $= (462 \times 8 \times 30)$ cu. in.

Surface area of cistern $= (\frac{22}{7} \times \frac{49}{4})$ sq. ft.;

$\therefore$ surface of water is lowered $\left(\frac{462 \times 8 \times 30}{1} \times \frac{7}{22} \times \frac{4}{49} \times \frac{1}{144}\right)$ in.

310. Square on diagonal = twice area of square = 55 ac.

Diagonal $= \sqrt{55 \times 4840}$ yd.

311. $3\frac{1}{2}\%$ per annum $= 2\frac{5}{8}\%$ for 9 mo., etc.;

$\therefore 2\frac{5}{8}\%$ of *A*'s share $= 2\frac{13}{16}\%$ of *B*'s share

$= 3\frac{3}{8}\%$ " *C*'s "

321. Height water will rise $= \frac{\frac{4}{3} \times \frac{22}{7} \times 2^3}{\frac{22}{7} \times 4^2}$ ft.

336. \$1 in 1 year amounts to $\sqrt[3]{\dfrac{980.0344}{800}}$.

337. See solution of No. 145, page 106

340. $\text{Mean breadth} = \dfrac{2\text{ ft. }5\text{ in.} + 2\text{ ft. }7\text{ in.}}{2} = 2\text{ ft. }6\text{ in.}$

$\text{Mean thickness} = \dfrac{1\text{ ft. }8\text{ in.} + 1\text{ ft. }6\text{ in.}}{2} = 1\text{ ft. }7\text{ in.}$

$\text{Volume} = (\frac{61}{3} \times \frac{5}{2} \times \frac{19}{12})\text{ cu. ft.}$

341. $\text{Slant height} = \dfrac{\text{twice the area of curved surface}}{\text{circumference of face}}$.

343. $\text{Volume} = (\frac{11}{2} \times \frac{11}{3} \times \frac{6}{1})\text{ cu. ft.}$

$= (121 \times 28.315312)\text{ cu. dcm.}$

and 1 cu. dcm. = 1 l.

347. $\frac{11}{12}$ of 123.274 gr. of pure gold are worth \$4.8665.

$\frac{9}{10}$ of 6.45161 × 15.43235 gr. of pure gold are worth

$\$\left(4.8665 \times \dfrac{12}{11 \times 123.274} \times \dfrac{9 \times 6.45161 \times 15.43235}{10}\right)$.

348. 6% per annum = $1\frac{1}{2}$% for 3 mo.;

$\therefore$ $116\frac{1}{2}$% of cost = $97\frac{1}{2}$% of \$1500.

349. \$500 in 3 yr. amounts to \$595.508.

\$1 " 1 " " " $\sqrt[3]{\dfrac{595.508}{500}} = \sqrt[3]{\$1.191016}$.

350. 16 cows + 120 sheep cost \$465.

$(\frac{215}{200} \times 16)$ " $+ (\frac{106}{100} \times 120)$ sheep cost \$496.50.

Eliminate as in an equation of two unknowns.

351. If the whole amount was invested in the 4 per [illegible]s, the income = $\$(\frac{5500}{1} \times \frac{1}{84} \times \frac{4}{1}) = \$\frac{5500}{21}$, or $\$\frac{100}{21}$ less than the actual income.

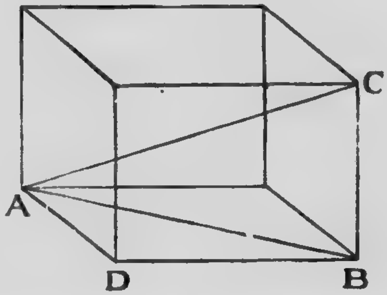
\$$102\frac{1}{2}$ earns \$5; $\therefore$ \$84 earns $\dfrac{84}{102\frac{1}{2}}$ of \$5, or $\$4\frac{4}{21}$.

\$$\frac{4}{41}$ is the gain in income by investing \$84 in the 5 per cents.
$\frac{100}{21}$ " " " " " " " \$$\left(\frac{84}{1}\times\frac{41}{4}\times\frac{100}{21}\right)$,
or \$4100 in the 5 per cents.

352. *A* and *B* receive \$960 + \$1370.
A and *B* pay out \$470 + \$330; ∴ they divide \$1530;
∴ *A* gets $\frac{11}{18}$ of \$1530 and already has \$960 − \$470.

353. 7% per annum is $1\frac{3}{4}$% for 3 mo.
\$$98\frac{1}{4}$ is realized when the amount of the note is \$100.

354.



Volume = (27 × 1728 × 2) cu. in.
Side = $(3\times 12\times\sqrt[3]{2})$ in.
$AB^2 = 2BD^2 = 2BC^2$.
$AC^2 = AB^2 + BC^2 = 3BC^2$.
AC = $(\sqrt{3}\times BC)$ in.
= $(\sqrt{3}\times 36\times\sqrt[3]{2})$ in.

357. \$128 invested in bank stock earns \$8 less the income tax.
Suppose \$100 invested in city property.
Repairs, etc., cost \$2; taxes $18\frac{1}{2}$ mills on \$90 or \$$1.66\frac{1}{2}$;
∴ the net income is \$$6.33\frac{1}{2}$ on an investment of \$100.
The net income is \$78.56 on an investment of
\$$\left(\frac{100}{1}\times\frac{200}{1267}\times\frac{7856}{1000}\right)$.

Investments are in proportion of \$128 in bank stock to \$$\frac{157120}{1267}$ in city property.

359. Income from the 7% stock is \$$\left(\frac{7}{1}\times\frac{1500}{115}\right)$.
∴ " " " 6% " " \$$\left(\frac{7}{1}\times\frac{1500}{115}\times\frac{4}{3}\right)$;
∴ amount invested in 6% " " \$$\left(\frac{7}{1}\times\frac{1500}{115}\times\frac{4}{3}\times\frac{1}{6}\times\frac{104}{1}\right)$.

361. Area of surface is now $\frac{110}{100}\times\frac{115}{100}$ or $\frac{1265}{1000}$ of former area;
∴ thickness will be $\frac{1000}{1265}$ of the former thickness;
∴ the thickness is diminished by $\frac{265}{1265}$ " " " "

362. Mortgage of \$800000 at 5% costs \$40000 per annum.
Shares " \$1200000 " 4% " \$48000 " " .
∴ the company requires to earn \$88000 over expenses;
∴ 45% of earnings per year = \$88000.

363. 299 times (76 men + 59 boys) would do in 1 da. what 76 men and 59 boys do in 299 da.

557 times (40 men + 33 boys) would do in 1 da. what 40 men and 33 boys do in 557 da.;

∴ 299 (76 men + 59 boys) = 557 (40 men + 33 boys).
Whence 740 boys do as much as 444 men.
15 " " " " 9 "

364. He sells 1 share of \$100 of 3 per cents at $94\frac{1}{2}$ and re-invests in $8\frac{1}{2}$ per cents.

Income is $\$\left(\frac{94\frac{1}{2}}{225} \text{ of } 8\frac{1}{2}\right)$, or $\$\frac{3213}{900}$;

∴ he gains $\$(\frac{3213}{900} - 3)$, or $\$\frac{513}{900}$ by transferring 1 share of 3 per cents.

He gains \$57 by transferring $(\frac{57}{1} \times \frac{900}{513})$ shares of \$100 each.

366. When *A* and *B* begin partnership, *A* invests \$20000 for 3 yr., which is equal to an investment of \$60000 for 1 yr.

B is to get half the profits; ∴ he is treated as if he had \$60000 invested for 1 yr.

In addition *A* puts in \$16000 more for the last two years; this is equivalent to \$32000 for 1 yr.;

∴ \$192000 invested for 1 yr. earns \$28500.

367. Area of whole surface is area of base + area of the four equilateral triangular faces. Side of triangle is 12. in.

368. If 3 more men were employed there would be twice as many men as women.

3 men would earn \$33.30 per week.

The weekly wage would then be \$662.40 and 2 men and 1 woman would earn \$28.80 per week;

∴ there would be $\frac{66240}{2880}$ women, or 23 women.

372. Sells $\frac{1}{2}$ at $36\frac{1}{2}$ in. per yard;

∴ he sells $(\frac{5183}{2} \times \frac{36}{1} \times \frac{2}{73})$ yd., or 2556 yd.

Sells $\frac{1}{2}$ at $35\frac{1}{2}$ in. per yard; sells $(\frac{5183}{2} \times \frac{36}{1} \times \frac{2}{71})$, or 2628 yd.

Altogether he sells 5184 yd. at 25% advance;

∴ he realizes price of $(\frac{5184}{1} \times \frac{125}{100})$ yd., or 6480 yd.;

∴ he gains the price of 6480 − 5183, or 1297 yd.;

∴ 1297 yd. cost $2594.

374. Cost price of first farm $= \frac{7}{8}$ selling price.

" " " second " $= \frac{7}{6}$ " "

∴ cost price of the two $= (\frac{7}{8} + \frac{7}{6})$ selling price of one farm.

$= 2\frac{1}{24}$ " " " " "

∴ loss $= \frac{1}{24}$ selling price of one farm = $250.

Selling price of one farm = $6000.

376. He realized the price of 1500 felt hats on the whole sale.

He should have realized the price of 1680 hats;

∴ he lost the price of 180 hats when the price was reduced from 140% to 90%.

50% of the price of one hat is lost on each by the reduction;

∴ price of 180 hats is lost on the sale of 360 hats below cost.

379. 1 share costs $108\frac{3}{8} + \frac{1}{8} = 108\frac{1}{2}$.

1 share realizes $110\frac{5}{8} - \frac{1}{8} = 110\frac{1}{2}$;

∴ on 1 share he increases his capital by the dividend + $2, or $5.25.

$325.50 is increase from 62 shares of $100 each.

380. 1400 t. at $5.15 cost $7210.

1400 t. sold for $8596.53\frac{11}{13}$.

He realized $\$(\frac{515}{100} \times \frac{7}{6})$ by selling 1 t.

He realized $8596.53\frac{11}{13}$ by selling $\left(8596.53\frac{11}{13} \div \frac{515 \times 7}{100 \times 6}\right)$ t.

But in actual weight he sold 1400 t.;

$$\therefore \left(8596.53\frac{11}{13} \div \frac{515 \times 7}{100 \times 6}\right) \text{ t. weigh } 2000 \times 1400 \text{ lb.}$$

$$1 \text{ t. weighs } \left(\frac{2000 \times 1400}{1} \times \frac{13}{111755} \times \frac{515 \times 7}{100 \times 6}\right) \text{ lb.}$$

381. $(1.2)^4$ times capital $-$ \$500$\{(1.2)^3+(1.2)^2+(1.2)+1\}$ $=$ \$18052,

whence 2.0736 times capital $=$ \$20736.

382. Amount received from sale of silk exceeds amount invested in flour by \$340;

∴ commission on \$340 sales is \$8.50.
" " \$100 " " \$2.50.

383. 8000 m. $=$ 26400 ft., or 10 m. $=$ 33 ft.;

∴ 1000 cu. m. $=$ 35937 cu. ft.
1000000 kg. $=(\frac{35937}{216}\times\frac{13440}{1})$ lb.

384. He sold 64 lb. at $58\frac{1}{2}$c., realizing \$37.44.

Cost price $=\frac{5}{6}$ of \$37.44 $=$ \$31.20.
28 lbs. at 20c. $=$ \$5.60;
∴ if both qualities had cost the same, 64 lb. would cost \$25.60.

385. Volume of cube $=\left(\frac{22}{7}\times\frac{7^2}{16^2}\times\frac{1}{16}\times\frac{100000}{1}\right)$ cu. in.

Side of cube $=\left(\sqrt[3]{\frac{22}{7}\times\frac{7^2}{16^2}\times\frac{1}{16}\times\frac{100000}{1}}\right)$ in.

386. *A* does $\frac{1}{2}$ the work in 12 hr.; ∴ *B* would do $\frac{1}{2}$ of it in 18 hr. and *C* in $\frac{180}{7}$ hr.

Then, if *B* and *C* each work 1 hr. they will do $\frac{1}{18}+\frac{7}{180}$, or $\frac{17}{180}$ of the balance;

∴ $\frac{17}{180}$ of balance is done in 2 hr.

387. Volume of brass $=$ 1728 cu. in.

Area of cross section of wire $=\left(\frac{22}{7}\times\frac{1}{44^2}\right)$ sq. in. $=\frac{1}{7\times 2\times 44}$ sq. in;

∴ a wire $(7\times 2\times 44)$ in. long will have a volume of 1 cu. in.;
∴ a wire $(7\times 2\times 44\times 1728)$ in. long will have a volume of 1 cu. ft.

388. The first cost $\frac{3}{4}$ of the second.

He sold the first for $\frac{5}{4}$ of $\frac{3}{4}$ of cost of second and the second for $\frac{9}{10}$ of its cost.

Total cost $= 1\frac{3}{4}$ of cost of second. Total selling price $= \frac{147}{80}$ of cost of second;

$\therefore$ he gained $\frac{7}{80}$ of cost of second on $1\frac{3}{4}$ of cost of second, or 5%.

389. 1095 lb. is the weight of 1 cu. ft.

1 oz. is the weight of $\left(\frac{1728}{1} \times \frac{1}{1095 \times 16}\right)$ cu. in.

Surface area of sheet weighing 1 oz. is 20 sq. yd.;

$\therefore$ thickness " " " 1 "

$$\text{is } \left(\frac{1728}{1} \times \frac{1}{1095 \times 16} \times \frac{1}{9 \times 20 \times 144}\right) \text{ in.};$$

$\therefore$ number of sheets in pile $1\frac{1}{2}$ in. high is

$$\frac{3}{2} \times \frac{1095 \times 16 \times 9 \times 20 \times 144}{1728}.$$

391. $\frac{1}{15}$ of mixture $= 9.4$ l.;

$\therefore$ mixture contains (9.4×14) l. of wine, in proportion of $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$, or $15 : 12 : 10$.

392. The insurance is $\frac{7}{8}$ value $+ \$100$.

" premium " $\frac{21}{800}$ value $+ \$3$;

$\therefore$ the loss would be $\frac{1}{8}$ value $- \$100 + \frac{21}{800}$ value $+ \$3$, or $\frac{121}{800}$ value $- \$97$.

Hence, $\frac{121}{800}$ of value $- \$97 = \$508$.

395. This depends on the well-known fact that the line joining the middle points of the sides of a triangle is parallel to the third side and equal to half of it; the area of the small triangle thus formed is one-fourth that of the main traingle.

397. In this case there are two equilateral triangles making up the rhombus.

399. On a value of $\$99\frac{1}{8}$ the premium is $\$\frac{7}{8}$.

" " " " $5551 " " " $\$\left(\frac{7}{8} \times \frac{8}{793} \times \frac{5551}{1}\right)$.

400. $4800 in 8 mo. earns as much as $38400 in 1 mo.

A gained $1200 and *C* gained $1800.

$1200 is gained by $38400 in 1 mo.

$1800 " " " $57600 " 1 "

$1800 " " " $6000 " $\frac{57600}{6000}$ mo.

Let $x = B$'s stock.

Then it earns as much as $10x$ in 1 mo.;

$\therefore \frac{10x}{38400}$ of \$1200 + \$$x$ = \$7000.

See solution of No. 433.

401. See solution of No. 63, page 202.

405. Let A B C D be the platform. Through A draw A E parallel to D C and A F perpendicular to B E. The triangle A B E is isosceles and A F is $(4\sqrt{5})$ ft. in length.

408. The third boat gains 100 yd. on the second in 6 min. or $16\frac{2}{3}$ yd. per minute.

The third boat gains 200 yd. on the first in 8 min., or 25 yd. per minute;

$\therefore$ the second boat gains $8\frac{1}{3}$ yd. per minute on the first boat;

$\therefore$ " " " " 100 yd. in 12 min. " " "

409. The income tax is $\frac{1}{10}$, leaving a residue of $\frac{9}{10}$.

Rate is $\frac{1}{8}$, leaving $\frac{7}{8}$ of the residue, or $\frac{7}{8}$ of $\frac{9}{10}$;

$\therefore \frac{7}{8}$ of $\frac{9}{10}$ of gross product of estate = \$1612.

411. Rate during 1st quarter = 30 mi. per hour.

" " 2nd " = 35 " " "

" " 3rd " = 40 " " "

" " 4th " = 40 " " "

$\therefore$ average $= \frac{145}{4}$ mi., or $36\frac{1}{4}$ mi.

412. 450 yd. are done in $(100 \times 30 \times 9)$ hr.

550 " " " " $\left(\frac{100 \times 30 \times 9}{450} \times \frac{550}{1}\right)$ hr.

The men work 10 hr. a day for 20 da.;

$\therefore$ he will require $\left(\frac{100 \times 30 \times 9}{450} \times \frac{550}{8} \times \frac{1}{10 \times 20}\right)$ men, or 165 men.

He must add 65 men to his staff.

413. L. C. M. $= 17 \times 5929 = 17 \times 49 \times 121$.

Difference $= 17 \times 72$, and $121 - 49 = 72$;

$\therefore$ numbers are 17×49 and 17×121.

414. 5000 bu. at $1.20 = $6000.
Commission = $500;
∵ he invests $5500 at 4% commission;
∴ commission for selling is $280.

415. 8 lb. cost $5; ∴ 1 lb. costs 62½c.
5 lb. sell for $3; ∴ 1 lb. sells for 60c.;
∴ he loses 2½c. on an outlay of 62½c.
Again, 3 lb. sell for $2, or 1 lb. for 66⅔c.;
∴ he gains 4⅙c. on an outlay of 62½c.

418. Area of walls and ceiling of first room is 1184 sq. ft. Openings = 94 sq. ft., leaving 1090 sq. ft. to be plastered.
Second room has 70¼ sq. ft. of openings;
∴ area of walls and ceiling of second room is 1160¼ sq. ft.
Let x = length of second room,
then area of walls and ceiling = $(40x + x^2)$ sq. ft.;
∴ $x^2 + 40x = 1160\frac{1}{4}$, whence $x = 19\frac{1}{2}$.

420. ½ of land cost $5000 and sold for $7500.
The remainder = $\{1 - (\frac{1}{2} + \frac{2}{5})\}$ of land = $\frac{1}{10}$ of land and cost $1000.
" " sold for 60% of cost, or $600.
he total amount realized was 145% of $10000;
.. the amount realized from the ⅖ at $40 per acre was $6400.

421. $300 at 1% for 2 yr. earns $6 interest;
∴ at the old rate the amount would be $405 − $6 or $399;
∴ interest on $300 for 2 yr. is $39.

423. He has to pay interest on the original cost of the goods for 6 mo. at 8%, making total cost $291.20.

424. He pays interest $120, insurance $45, and taxes $88, leaving a net revenue of $227 from the rent of house;
∴ he has $2282 left from income, with $500 exempt;
∴ " " $1782 " " taxable portion of income.
Income tax is 10 mills or 1%;
∴ 99% of taxable portion = $1782.
Taxable portion of income is $1800.

425. Life insurance premiums are paid in advance.

The value of the payments for premiums at the end of the first ten years would be

$\$[80\{(1.06)^{10}+(1.06)^9+(1.06)^8+ - - - +(1.06)^2+(1.06)\}]$, or \$1117.73; this sum in 10 yr. at 8% $=\$\{1117.73(1.08)^{10}\}$;

∴ its value at the end of insurance period = \$2413.10.

The value of the payments during the second period of 10 yr. is $\$[80\{(1.08)^{10}+(1.08)^9+ - - - +(1.08)^2+(1.08)\}]$, or \$1251.64;

∴ total value of payments would be \$3664.74.

426. The spectator is equidistant from the marksmen and the target;

∴ the report of the gun and the report of the bullet striking the target would require the same time to reach him.

But the report of the bullet striking the target is $2\frac{2}{3}$ sec. later than the report of the gun;

∴ the bullet requires $2\frac{2}{3}$ sec. to reach the target.

Velocity of bullet is 800 yd. in $2\frac{2}{3}$ sec.

" " sound " 800 " " $(4\frac{28}{33}-2\frac{2}{3})$ sec.

428. On $\frac{2}{5}$ of goods he realizes $(\frac{2}{5}\times\frac{4}{3})$ of cost, or $\frac{8}{15}$ cost.

Altogether he realizes $\frac{6}{5}$ cost (gains 20%);

∴ on the balance sold he realizes $(\frac{6}{5}-\frac{8}{15})$ cost, or $\frac{2}{3}$ cost.

He sells for $(\frac{4}{3}\times\frac{215}{200})$ cost;

∴ he has to sell $\frac{2}{3}\div(\frac{4}{3}\times\frac{215}{200})$ to realize $\frac{2}{3}$;

∴ he sells $\frac{2}{5}+\frac{20}{43}$. The balance is unsaleable.

429. Rate of gain on first part $=\frac{3}{8}$ of cost

$=\frac{3}{11}$ of selling price;

∴ gain $=\frac{3}{11}$ of \$330 = \$90.

Hence, loss on second part = \$15;

∴ loss on \$135 is \$15, and loss % is $11\frac{1}{9}$.

430. Investment at end of first year $=\frac{1}{4}$ of salary for first year

" " second " $=\frac{1}{4}$ of $\frac{5}{4}$ " " " "

$=\frac{5}{16}$ " " " "

" " third " $=\frac{1}{4}$ of $\frac{5}{4}$ of $\frac{5}{4}$ of salary for first year $=\frac{25}{64}$ of salary for first year.

The investment at end of first year amounts to $\frac{1}{3}(1.05)^2$ of salary for first year.

The investment at end of second year amounts to $\frac{5}{12}(1.05)$ of salary for first year;

∴ the total investment is $(\frac{147}{400}+\frac{7}{16}+\quad)$ of salary for first year;

∴ $(\frac{147}{400}+\frac{7}{16}+\frac{25}{48})$ of salary for t year = $795.50.

431. The selling price = $\frac{7}{8}$ of cost price.

Hence, $\frac{31}{36}$ of (cost price − 12c.) = $\frac{7}{8}$ cost price.

432. Suppose he borrows $120 and buys 1 share of $100 of stock.

This yields $7 income and sells for $119, but he pays an income tax of 20 mills, or 14c. on $7;

∴ he has ($119 + $7 − $.14) to repay ($120 + $6).

He loses 14c. on each share purchased.

433. *C* invests $3220 for 16 mo. and gains $644;

∴ $644 would be gained by 16 × $3220 in 1 mo.

$624 " " " " $\$\left(\frac{16\times 3220}{1}\times\frac{624}{644}\right)$ in 1 mo;

∴ *B*'s money is invested for $\left(\frac{16\times 3220}{1}\times [illegible] \times\frac{1}{4160}\right)$ mo.

$3220 × 16 gains $644 in 1 mo.

$100 " $\$\left(\frac{644}{3220}\ \frac{100}{\times 16}\right)$, or $\$\frac{5}{4}$ in 1 mo.

$100 gains $10 in 8 mo ;

∴ $110 is the amount of $100 in 8 mo.

$2354 " " " $\$(\frac{100}{110}$ of $\frac{2354}{1})$ in 8 mo.

435. *A* gains $1\frac{1}{2}$ mi. on *C* in 40 min., or $2\frac{1}{4}$ mi. per hour.

A " $\frac{5}{8}$ " " *B* in 50 " " $\frac{3}{4}$ " " "

∴ *B* " $1\frac{1}{2}$ " " *C* in 1 hr.;

∴ *B* " $\frac{7}{8}$ " " *C* in 35 min.

436. Value of house + value of lot = $2900.

$\frac{7}{8}$ " " " + $\frac{9}{8}$ " " " = $3000,

whence value of house was $240 . Present value $2100

It is insured for $\frac{9}{7}$ of $2100 at $\frac{7}{8}$%.

437. 4 premiums of $1\frac{1}{4}\%$ each $= 5\%$ on $\frac{3}{5}$ of the value, or 3% of the value;

$\therefore$ the owner netted $60\% - 3\%$, or 57% and lost 43%.

439. Suppose he buys 1 share at 108, his income is \$6.

To receive income of \$6 from second stock he must invest $\$\frac{95 \times 6}{5}$, or \$114;

$\therefore$ an investment of (\$108 + \$114) yields an income of \$12.

440. Gain on first farm $= \frac{1}{7}$ of cost $= \frac{1}{8}$ of selling price.

Loss on second farm $= \frac{1}{12}$ of cost $= \frac{1}{11}$ of selling price;

$\therefore$ net gain $= (\frac{1}{8} - \frac{1}{11})$ of selling price of one farm $= \$300$.

441. In 1 hr. 20 min. *A* travels 5 mi. *B* overtakes *A* in 4 hr.;

$\therefore$ *B* travels 20 mi. in 4 hr.

B travels as far in 3 hr. as *A* in 4 hr.;

$\therefore$ when *B* has travelled 3 hr. after overtaking *A* he is 1 hr. ahead of *A*.

When *B* has travelled 12 hr. he is 4 hr. ahead of *A*.

He was 4 hr. overtaking *A*, $\therefore$ 16 hours on the journey.

443. He sells $7\frac{1}{2}$ pt. for 75% of 125% of cost, or $118\frac{3}{4}\%$ of cost.
“ “ 8 “ “ $(118\frac{3}{4} \times \frac{8}{1} \times \frac{2}{15})\%$ of cost.

444. 14500 bu. wheat at 65c. = \$9425.
Storage at $\frac{1}{2}$c. per bushel = 72.50
Other expenses = 18.50
\$9516.

$\$99\frac{1}{8}$ is covered by policy for \$100
\$9516 “ “ “ “ “ $\$(\frac{100}{1} \times \frac{8}{793} \times \frac{9516}{1})$.

446. *A* works $(7 \times 4\frac{1}{2} + 8 \times 8)$ hr. $= 95\frac{1}{2}$ hr.
B “ $(8 \times 4\frac{1}{2} + 9 \times 8)$ “ $= 108$ “ $= 94\frac{1}{2}$ hr. at *A*'s rate
C “ $(9 \times 4\frac{1}{2} + 10 \times 8)$ “ $= 120\frac{1}{2}$ “ $= 93\frac{13}{18}$ “ *A*'s “
D “ $(10 \times 4\frac{1}{2} + 11 \times 8)$ “ $= 133$ “ $= 93\frac{1}{10}$ “ *A*'s “
$\therefore$ *A* in $376\frac{74}{90}$ hr. would earn \$169.57;
$\therefore$ *A* “ 1 “ “ “ 45c.;
$\therefore$ *B* “ 1 “ “ “ $\frac{7}{8}$ of 45c., etc.

447. Volume of ice $=(\frac{22}{7}\times\frac{49}{4}\times\frac{9}{1})$ cu. ft.

Volume of water $=(\frac{22}{7}\times\frac{49}{4}\times\frac{9}{1}\times\frac{10}{11})$ cu. ft.

Weight $=(\frac{22}{7}\times\frac{49}{4}\times\frac{9}{1}\times\frac{10}{11}\times\frac{125}{2})$ lb.

448. Diameter of cylinder is 2 ft.

Volume of cylinder is $(\frac{22}{7}\times\frac{1}{1}\times\frac{2}{1})$ cu. ft., or $\frac{44}{7}$ cu. ft.

Volume of cube $=8$ cu. ft.

Volume of circular sheet is $1\frac{5}{7}$ cu. ft.

$$(\pi r^2\times\tfrac{2}{11})\text{ cu. in.}=\frac{1728\times 12}{7}\text{ cu. in.}$$

$$r^2=\frac{1728\times 12}{7}\times\tfrac{11}{2}\times\tfrac{7}{22},$$ whence radius is 72 in.

449. Suppose he buys 1 bu. of wheat at \$1.00. He sells for \$1.28$\frac{4}{7}$.

Then he buys $\frac{1}{3}$ bu. of wheat at $\frac{3}{5}$ of \$1.00;

$\therefore$ 1$\frac{1}{3}$ bu. cost \$1.20 and sell for $\frac{4}{3}$ of \$1.28$\frac{4}{7}$;

$\therefore$ on an outlay of \$1.20 he gains 51$\frac{3}{7}$c.

450. Cross-section area of bore $=(\frac{22}{7}\times\frac{1}{4})$ sq. in.

" " " pipe $=(\frac{22}{7}\times\frac{17}{16}\times\frac{17}{16})$ sq. in.;

$\therefore$ " " material $=\left(\frac{3179}{7\times 128}-\frac{11}{14}\right)$ sq. in.

$$\text{Volume of material}=\left(\frac{2475}{7\times 128}\times\frac{64\times 12}{1728}\right)\text{ cu. ft.}$$

$$\text{Weight}=\left(\frac{2475}{7\times 128}\times\frac{64\times 12}{1728}\times\frac{11.52}{1}\times\frac{1000}{1}\times\frac{1}{16}\right)\text{lb.}$$

451. The diagonal of the field $=2200$ yd.

As the length is twice the breadth,

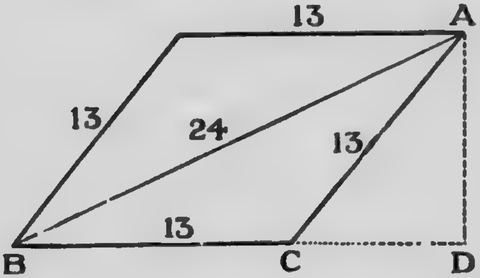
$\therefore$ (number of units in twice breadth)^2 + (number of units in breadth)$^2=2200^2$;

$\therefore$ 5 times (number of units in breadth)$^2=2200^2$,

and number of units in breadth $=440\sqrt{5}$.

Hence, area $=(440\sqrt{5}\times 880\sqrt{5})$ sq. yd.

452. Area is found by finding area of the triangles.



Drop a perpendicular AD on BC produced.

$BC^2 + CA^2 + 2BC\ CD = AB^2.$

$13^2 + 13^2 + 26CD = 24^2.$

Whence $CD = 9\frac{2}{13}.$

$AD^2 = AB^2 - BD^2 = 24^2 - (22\frac{2}{13})^2.$

453. Area of bottom of the well $= (\frac{22}{7} \times \frac{49}{1})$ sq. ft.

Volume to raise surface 5 ft. $= (\frac{22}{7} \times \frac{49}{1} \times \frac{5}{1} \times \frac{1728}{1})$ cu. in.

Area of cross-section of pipe = area of six equilateral triangles, each side of which is 1 in. $= \frac{3\sqrt{3}}{2}$ sq. in.

Length of pipe to raise the surface 5 ft.

$$= \left(\frac{22}{7} \times \frac{49}{1} \times \frac{5}{1} \times \frac{1728}{1} \times \frac{2}{3\sqrt{3}}\right) \text{in.}$$

$$\text{Number of hours} = \left(\frac{22}{7} \times \frac{49}{1} \times \frac{5}{1} \times \frac{1728}{1} \times \frac{2}{3\sqrt{3}} \times \frac{1}{12 \times 5280} \times \frac{1}{6}\right).$$

454. Cost of 1 share of \$100 is \$$(116\frac{7}{8} + \frac{1}{8})$, or \$117.

\$$(97\frac{3}{8} + \frac{1}{8})$ buys one share of 4% stock.

\$117 " $\frac{6}{5}$ shares " "

Net income is \$$\{(3\frac{1}{2} + \frac{6}{5} \text{ of } 4) \times \frac{985}{1000}\}$ = \$8.1755 when \$117 is invested in each kind of stock.

Net income is \$163.51 when $\frac{163.51}{8.1755} \times \frac{117}{1}$ is invested in each.

First stock realizes \$$(\frac{2340}{117} \times 116\frac{7}{8})$, or \$2337.50.

Second " " \$$\left(\frac{2340}{97\frac{1}{2}} \times 97\frac{3}{8}\right)$, or \$2337;

∴ he invests \$4838.01 in 10% at 158.

Net income will be \$$\left(\frac{4838.01}{158} \times \frac{10}{1} \times \frac{980}{1000}\right)$.

455. Amount of sales $= \$10400$.
Total commission $= \$700$.
Amount invested $= \$9700$.
Commission on $\$9700$ at $1\% = \$97$;

$\therefore$ his commission would be $\$603$ if rate for buying were same as rate for selling.

Commission on $(\$10400 + \$9700) = \$603$.
" " $\$100 = \$3$.

456. Compound interest for 2 yr. at $8\% = (1.08)^2$.
Compound interest for 2 yr. at 8% added half-yearly $= (1.04)^4$.
Difference in interest on $\$1$ is $\$\{(1.04)^4 - (1.08)^2\} = \$.00345856$.

457. At 5 mi. per hour he could walk $1\frac{1}{4}$ mi. past the place.
At 3 " " " he would be $1\frac{1}{4}$ mi. short of the place;
$\therefore$ the difference in distance walked is $2\frac{1}{2}$ mi.;
$\therefore$ " " " rate " is 2 mi. per hour;
$\therefore$ he walks $\frac{2\frac{1}{2}}{2}$, or $1\frac{1}{4}$ hr. at 5 mi., or $6\frac{1}{4}$ mi.
But this puts him $1\frac{1}{4}$ mi. past the place;
$\therefore$ the place is 5 mi. distant;
$\therefore$ he should walk $\frac{5}{1\frac{1}{4}}$ or 4 mi. per hour to arrive just on time.

PART III.—SUPPLEMENT

CASTING OUT NINES

1. Any number divided by nine will leave the same remainder as the sum of its digits divided by nine.

This will be evident from the following example:—

$$\begin{aligned}\frac{6783}{9} &= \frac{6000}{9}+\frac{700}{9}+\frac{80}{9}+\frac{3}{9}\\ &=(666+\tfrac{6}{9})+(77+\tfrac{7}{9})+(8+\tfrac{8}{9})+\tfrac{3}{9}\\ &=(666+77+8+\tfrac{6}{9}+\tfrac{7}{9}+\tfrac{8}{9}+\tfrac{3}{9})\\ &=751+\frac{6+7+8+3}{9}.\end{aligned}$$

This check will not detect such an error as the interchange of digits, nor any error not affecting the sum of the digits.

2. To check multiplication by casting out the nines.

This test may be given in the form of the following rule:—

Divide the sum of the digits in the Multiplicand by 9, *and set down the remainder. Divide the sum of the digits in the Multiplier by* 9, *and set down the remainder. Multiply the two remainders together, divide the result by* 9, *and set down the remainder. If the process be correct, this remainder will be the same as the remainder obtained by taking the sum of the digits in the Product and dividing it by* 9.

For example, if we multiply 76371 by 854 the product is 65220834.

Sum of digits in Multiplicand = 24,
and 24 ÷ 9 gives remainder 6.
Sum of digits in Multiplier = 17,
and 17 ÷ 9 gives remainder 8.

First remainder × second remainder = 48,
and 48 ÷ 9 gives remainder 3.

Sum of digits in the Product = 30,
and 30 ÷ 9 gives remainder 3.

This so-called proof is defective as a proof in the following, as it fails to detect errors in the product—

(*a*) If the order of figures in the product is misplaced, as 37 for 73.

(*b*) If errors be made which counterbalance each other, as 35 written for 62, the sum of the digits in each case being the same.

(*c*) If 9 be written for 0, or 0 for 9, or either be omitted or inserted too often.

ARITHMETICAL COMPLEMENT

3. The arithmetical complement of a number is defined to be the difference between any given number and the unit of the next superior order; thus 6 is the arithmetical complement of 4, 47 of 53, 8468 of 1532, and so on, being the differences respectively of 4, 53, 1532, and 10 100, 10000, the next superior units of these numbers. Conversely, also, 4, 53, 1532 are the arithmetical complements of 6, 47, 8468 respectively.

The arithmetical complement of a number may be found by the following rule:—

Begin at the left hand and subtract every figure from 9 *until the last; subtract that from* 10.

The arithmetical complement may be used to find the difference between two numbers, thus: if 239 be subtracted from 576 the remainder is 337. But if 761, the arithmetical complement of 239, the less number, be added to 576, the greater, the sum will be 1337, one unit (1000 in this case) of the next superior order greater than the difference of the two numbers. By removing this unit, the number will be left equal to the difference of 239 and 576; so that the difference of the two numbers can be found by addition. The arithmetical complement may be written thus $\bar{1}761$, with the subtractive unit on the left, which when added to 576, the sum will be 337, the additive and subtractive units being together equal to zero.

This method is employed with great advantage to find the aggregate of several numbers when some of them are additive and some subtractive. Thus, if we have—

$$3795 - 1532 - 2019 + 8759 - 5104 + 307,$$

we arrange them as follows:—

	3795
A. C. of 1532 is	$\bar{1}8468$
" 2019 "	$\bar{1}7981$
	8759
" 5104 "	$\bar{1}4896$
	307
	4206

the aggregate required.

4. To Square any Number.

$$78^2 = 80 \times 76 + 2^2 = 6084$$
$$34^2 = 30 \times 38 + 4^2 = 1156$$
$$75^2 = 80 \times 70 + 5^2 = 5625$$
$$113^2 = 110 \times 116 + 3^2 = 12769$$

Add to and subtract from the given number such a number as is necessary to form an exact number of tens. Multiply the resulting number by the number of tens thus found and to the product add the square of the number added and subtracted. The sum is the square of the given number.

5. To multiply using Arithmetical Complements when there are the same number of figures in the Multiplier and the Multiplicand.

Ex. Multiply 987 by 994.

A. C. of multiplicand = 13
A. C. of multiplier = 6
6 × 13 = 78; 078
987 − 6 = 981; 981078 = product required.

Hence, *multiply the A. C. of the multiplicand by the A. C. of the multiplier. Set down the product, filling up as many places to the left with naughts as will make the number of places equal to the number of figures in the multiplicand. Subtract the A. C. of the multiplier from the multiplicand and set the remainder to the left of the figures already placed. The result will be the product required.*

6. Tests of Exact Divisibility.

(*a*) A number is exactly divisible by 2 if its right-hand figure is zero or an even digit.

(*b*) A number is exactly divisible by 4 if its two right-hand figures are zeros or express a number exactly divisible by 4.

(*c*) A number is exactly divisible by 8 if its three right-hand figures are zeros or express a number exactly divisible by 8.

(*d*) A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3.

(*e*) A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

(*f*) A number is exactly divisible by 5, 25 or 125 if the number ends in 1, 2 or 3 zeros, or if the number expressed by the right-hand figure, or by the two, or by the three right-hand figures is exactly divisible by 5, 25 or 125 as the case may be.

(*g*) A number is exactly divisible by 11 when the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either 0 or exactly divisible by 11. Thus, 24794 and 829191 are exactly divisible by 11.

CUBE ROOT

7. When a number is multiplied by itself twice, the result is called the **Cube** of the number. Thus 27 is the cube of 3, and 216 is the cube of 6.

8. The **Cube Root** of a given number is that number whose cube is equal to the given number.

Thus the Cube Root of 343 is 7, because the cube of 7 is 343.

The symbol $\sqrt[3]{\ }$, placed before a number, denotes that the cube root of that number is to be taken; thus $\sqrt[3]{125}$ is read "the cube root of 125."

9. A number which has an integer for its cube root is called a **Perfect Cube.**

10. To find the Cube Root of a perfect cube, greater than 1000, we proceed by a rule which we shall now explain.

Ex. To find the Cube Root of 91125.

$$
\begin{array}{rrr|r}
 & 4 & & 91|125 \\
 & & & 64 \\
\hline
12 & 5 & 4800 & \\
 & & 625 & 27125 \\
\cline{3-3}
 & & 5425 & 27125 \\
\cline{4-4}
\end{array}
$$

First divide the number 91125 into two periods by drawing a line marking off *three* figures on the right.

Then take the nearest perfect cube not greater than 91, which is 64, and set down its cube root, which is 4, in a line with 91125, and some way to the left. This is the first figure of the root.

Then subtract 64 from 91, and to the remainder attach the second period, 125.

Now place three times the first figure of the root, 12, to the extreme left, and three times the square of the first figure of the root, 48, with two zeros annexed to it, just on the left of the 27125.

Divide 27125 by 4800, and set the quotient, 5, midway between 12 and 4800. Then read 12 5 as 125; multiply this by 5; put the result, 625, under the 4800; add to it the 4800; this gives 5425; multiply this by 5; put the result, which is 27125, under the first remainder; subtract, and as there is no remainder, the process is complete, and the root is 45.

Examples I

Find the Cube Roots of

1. 4096.
2. 32768.
3. 74088.
4. 493039.
5. 614125.
6. 262144.
7. 39304.
8. 389017.
9. 778688.
10. 970299.
11. 59319.
12. 250047.

Next, let us take the case in which the cube root has three figures and extract the cube root of 428661064.

	7		428|661|064
			343
21	5	14700	85661
		1075	
		15775	78875
		25	
225	4	1687500	6786064
		9016	
		1696516	6786064

We separate the number 428661064 into three periods, and take the nearest perfect cube not greater than 428, which is 343, and we set down its cube root, which is 7. We then subtract 343 from 428, and annex to the remainder 661, the second period.

Then we set down three times 7, which is 21, and three times the square of 7, which is 147, and annex two zeros to it.

Then we divide 85661 by 14700, which gives the quotient 5, and this we put down midway between 21 and 14700.

Then we multiply 215 by 5, which gives 1075; we add this to 14700; we multiply the result, 15775, by 5; and subtract the product, 78875, from 85661; and to the remainder we annex the third period, 064.

We then set down three times 75, which is 225,
and three times the square of 75, which is 16875.

N.B. –*This last result can be obtained by setting the square of 5, the second figure of the root, under the second divisor, and adding the three numbers coupled by the bracket.*

We then annex two zeros to 16875 and repeat the process explained above to find 4, the third figure of the cube root, which is in this case 754.

Examples II

Find the Cube Roots of

1. 14706125.
2. 149721291.
3. 28934443.
4. 300763000.
5. 2097152.
6. 5735339.
7. 99252847.
8. 1092727.
9. 16777216.
10. 194104539.
11. 84027672.
12. 130323843.
13. 322828856.
14. 354894912.
15. 700227072.
16. 134217728.
17. 122615327232.
18. 673373097125.

11. To extract the Cube Root of a Decimal Fraction.

In order that a Decimal Fraction may be a Perfect Cube, it must be of the 3rd, 6th, 9th . . . . order, the index of the order being some multiple of 3.

We then proceed in the following way :—

Ex. 1. To find the Cube Root of .343.

$$\sqrt[3]{.343}=\sqrt[3]{\frac{343}{1000}}=\frac{7}{10}=.7.$$

Ex. 2. To find the Cube Root of .039304.

$$\sqrt[3]{.039304}=\sqrt[3]{\frac{39304}{1000000}}=\frac{34}{100}=.34.$$

Ex. 3. To find the Cube Root of .012812904.

$$\sqrt[3]{.012812904}=\sqrt[3]{\frac{12812904}{1000000000}}=\frac{234}{1000}=.234.$$

12. To extract the cube root of an integer or decimal expression to a particular place of decimals, in the given expression, we must take *three times the number* of decimal places required.

Thus, to find the cube root of 4.23 accurately to three places of decimals, we extract the cube root of 4.230000000, making the given expression a decimal of the *ninth* order. In working this example, we find the cube root of 4.230000000, *regarded as a whole number*, and mark off three decimal places in the result.

13. The Cube Root of a *Vulgar Fraction* may be found by taking the roots of the numerator and denominator, or by reducing the fraction to a decimal of the 3rd, 6th, 9th . . . . order, and proceeding as in Art. 12.

Examples III

Find the Cube Root of

1. .389017.
2. .048228544.
3. 27054.036008.
4. $\frac{1331}{1728}$.
7. $405\frac{28}{125}$.
5. $\frac{250}{686}$.
6. $5\frac{13}{343}$.

Find to three places of decimals the Cube Roots of

3. 5.
9. 576.
10. .121861281.
11. 15.926972504.
12. $\frac{5}{6}$.
13. $\frac{3}{4}$.
14. $\frac{1}{3}$.
15. $7\frac{3}{5}$.
16. $3\frac{1}{5}$.

14. The *fourth* root of a number is found by taking the square root of the square root of the number.

Thus $\sqrt[4]{4096}=\sqrt{64}=8.$

The *sixth* root of a number is found by taking the cube root of the square root of the number,

Thus $\sqrt[6]{64}=$ [illegible]

Examples IV

Find the Fourth Roots of

1. 531441. 2. 4100625. 3. 1575.2961.

Find the Sixth Roots of

4. 4826809. 5. 24794911296. 6. 282429.536481.

Examples V

1. The product of three equal numbers is 679151439. Find one of them.

2. There are three numbers, the second is twice the first, and the third is twice the second. The product of the three is 5000211000. Find the largest number.

3. Of three numbers the second is $\frac{1}{3}$ of the first and the third is twice the second. Their product is 178746. Find the numbers.

4. There are four numbers, the second being twice the first, the third twice the second, and the fourth twice the third. Their continued product is 585640000. Find the numbers.

5. Write down the squares of 8, 9, 10 and 11, and from these squares derive a rule for finding the square of any number when the square of the number next greater or next less to it is known.

6. The square of 5987 is 35844169. From this find the square of 5988 and of 5986.

7. Resolve the number 3456649728 into prime factors and from these determine the cube root of the given number.

8. Find the ninth root of 387420489.

15. The extraction of the cube root, by the ordinary rule, is a troublesome process, seldom used and easily forgotten. The following process is much simpler and more easily remembered.

Let a be an approximate value of the cube root of N, so that $\sqrt[3]{N}=a+x$, x being very small;

then $$N=(a+x)^3=a^3+3a^2x+3ax^2+x^3$$
$$=a^3+3ax(a+x), \text{ nearly, since } x \text{ is small.}$$

First suppose $N=a^3+3a^2x$;

$$\therefore\ x=\frac{N-a^3}{3a^2};$$

and $$\therefore\ a+x=\frac{N+2a^3}{3a^2};$$

therefore more nearly, $$N=a^3+3ax\frac{N+2a^3}{3a^2}$$

and $$x=\frac{N-a^3}{N+2a^3}a,$$

and, therefore, $$\sqrt[3]{N}=a+x$$
$$=\frac{2N+a^3}{N+2a^3}a.$$

Suppose we want to find the cube root of any number N. In the first place we find some number a whose cube is somewhere near the given number. Then the fraction,

$$\frac{2N+a^3}{N+2a^3}.a$$

will be a nearer approximation to the cube root than a itself was. When we have found this value, we can take this as a and repeat the process.

Thus, to find the cube root of 241.804367, we observe that 216, the cube root of 6, is nearest to 241. Hence the first value of a is 6.

Therefore, $$\frac{2N+a^3}{N+2a^3}.a$$
$$=\frac{699.608734}{673.804367}\times 6$$
$$=\frac{4197.652404}{673.804367}$$
$$=6.23, \text{ very nearly.}$$

On trying 6.23, we find it is correct.

Ex. Find the Cube Root of 47.

The nearest cube to 47 is that of 4.

Hence,
$$\frac{2N+a^3}{N+2a^3}.a$$
$$=\frac{94+64}{47+128}\times 4$$
$$=\frac{632}{175}$$
$$=\frac{2528}{700}$$
$$=3.61\text{, nearly.}$$

Next, take 3.61 for a, and substitute in the formula, and we get 3.6088261, which is correct to seven places of decimals.

Examples VI

Find to four places of decimals the Cube Root of

1. 12.
2. 30.
3. 225.
4. 375.
5. .9.
6. .08.
7. 9.27.
8. .587.
9. 8.76.

COMMISSION

16. Manufacturers and others frequently employ a third person to sell their goods or to buy goods for them. This third party is called a *Commission Merchant* or *Agent.* The pay he receives for his services is called *Commission.*

An *agent* receives a consignment of 400 head of cattle to sell. He is to be allowed 3 per cent. of the *gross proceeds* for making the sale, and to remit the *net proceeds* to the *consignor.* He sells the cattle at $42 per head.

The gross proceeds $= \$42\times 400 = \$16800$.

Agent's commission $= \$16800 \times \frac{3}{100} = \$504$.

The net proceeds $= \$16800 - \$504 = \$16296$.

The commission is $\frac{504}{16296}$, or $\frac{3}{97}$ of the net proceeds, and $\frac{3}{100}$ of the gross proceeds.

Suppose Mr. A. instructs the *agent* to buy for him 600 sheep at \$7.50 each and agrees to allow him 3 per cent. of the cost of the sheep for making the purchase.

$$\text{Total cost of sheep} = \$7.50 \times 600 = \$4500.$$
$$\text{Commission for buying} = \$4500 \times \tfrac{3}{100} = \$135.$$

Therefore Mr. A. must send the agent \$4500 + \$135, or \$4635. The commission is $\frac{135}{4635}$ or $\frac{3}{103}$ of the sum remitted, and $\frac{3}{100}$ of the sum invested.

PRESENT WORTH AND TRUE DISCOUNT

17. Suppose *A* owes *B* \$105, to be paid at the end of a year. If *A* be disposed to pay off the debt at once, the sum which he ought to pay should be such that, if put out at interest by *B*, it will amount at the end of a year to \$105. Suppose, further, that *B* can put out his money at 5 per cent. interest. Then, if he put out \$100 at interest, this is the sum which will amount at the end of a year to \$105.

Hence, \$100 is the sum which *A* ought to pay at once, and this is called the *Present Worth* of the debt, and is evidently such a sum as would, if put out at interest for the given time and rate, amount to the debt. The difference between the Debt and the Present Worth, which is in the case under consideration \$5, is called the Discount.

This kind of Discount is known as *Mathematical* or *True Discount*, and is to be distinguished from both Trade and Bank Discounts.

Its computation falls under the case of Simple Interest in which the Amount, Rate and Time are given to find the Interest or the Principal. In True Discount the Debt corresponds to the Amount, the Present Worth to the Principal and the True Discount to the Interest.

There are, however, a variety of problems which may arise in connection with True Discount, some of which are illustrated below.

Ex. 1. Thus, to find the Present Worth of \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 yr. at 5% = \$20;
∴ \$120 has for its present worth \$100;
$\therefore \$1 \text{ has for its present worth } \$\frac{100}{120};$

$$\therefore \$1781.40 \text{ has for its present worth } \$\frac{1781.40 \times 100}{120},$$

or \$1484.50;
∴ present worth required = \$1484.50.

Ex. 2. Find the Discount on \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 yr. at 5% = \$20;
∴ \$120 has for its discount \$20;
$\therefore \$1 \text{ has for its discount } \$\frac{20}{120};$

$$\therefore \$1781.40 \text{ has for its discount } \$\frac{1781.40 \times 20}{120}$$

= \$296.90.

Ex. 3. What was the debt of which the Discount for 8 months at 9 per cent. was \$44.46?

The interest on \$100 for 8 mo. at 9% = \$6;
∴ \$6 is the discount on \$106;
$\therefore \$1 \text{ is the discount on } \$\frac{106}{6};$

$$\therefore \$44.46 \text{ is the discount on } \$\frac{44.46 \times 106}{6}$$

= \$785.46.

Ex. 4. The interest on a certain sum of money for two years is \$50, and the discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

Since \$50 is the interest on a sum of money which sum
= (its present worth + its discount)
= (its present worth + \$45)
and \$45 is the interest on its present worth;
∴ \$5 is the interest on \$45;
$\therefore \$1 \text{ is the interest on } \$\frac{45}{5};$

$\therefore$ \$50 is the interest on $\$\frac{50 \times 45}{5}$, or \$450;
$\therefore$ \$450 is the sum required.

Again, the interest on \$45 for 2 yr. = \$5;
$\therefore$ the interest on \$45 for 1 yr. = $\$\frac{5}{2}$;
$\therefore$ the interest on \$1 for 1 yr. = $\$\frac{5}{45 \times 2}$;
$\therefore$ the interest on \$100 for 1 yr. = $\$\frac{100 \times 5}{45 \times 2} = \$5\frac{5}{9}$;
$\therefore$ the rate = $5\frac{5}{9}$%.

Note I.—*From the above it will be seen that the discount on any sum is the present worth of the interest of that sum for the same time and rate. Thus, \$45 is the present worth of \$50 for 2 yr. at a certain rate per cent.*

Ex. 5. If \$20 be allowed off a bill of \$420 due in 6 months, how much shall be allowed off the same bill due in 12 months?

The discount off \$420 for 6 mo. = \$20;
$\therefore$ the interest on \$400 for 6 mo. = \$20;
$\therefore$ the interest on \$400 for 12 mo. = \$40;
$\therefore$ the discount off \$440 for 12 mo. = \$40;
$\therefore$ the discount off \$1 for 12 mo. = $\$\frac{40}{440}$;
$\therefore$ the discount off \$420 for 12 mo. = $\$\frac{420 \times 40}{440} = \$38\frac{2}{11}$;
$\therefore$ the discount required = $\$38\frac{2}{11}$.

Note II.—*The student will observe that the discount is not proportioned to either the time or the rate.*

Ex. 6. If \$15 be the interest on \$115 for a given time, what should be the discount off \$115 for the same time?

The interest on \$115 = \$15;
$\therefore$ the discount off \$130 = \$15;
$\therefore$ the discount off \$1 = $\$\frac{15}{130}$;
$\therefore$ the discount off \$115 = $\$\frac{115 \times 15}{130} = \$13\frac{7}{26}$;
$\therefore$ the discount required = $\$13\frac{7}{26}$.

Ex. 7. If \$10 be allowed off a bill of \$110, due 8 months hence, what should be the bill from which the same sum is allowed as 4 months' discount?

\$10 is the discount off \$110 for 8 mo.;
∴ \$10 is the interest on \$100 for 8 mo.;
∴ \$10 is the interest on \$200 for 4 mo.;
∴ \$10 is the discount off \$210 for 4 mo.;
∴ the sum required = \$210.

Examples VII

1. Find the present worth of

(*a*) \$5520, due 4 yr. hence, at 5%.
(*b*) \$84.70, due $2\frac{1}{3}$ yr. hence, at 9%.
(*c*) \$615, due 1 yr. 4 mo. hence, at 7%.
(*d*) \$1120, due 16 mo. hence, at 5%.
(*e*) £618 2*s*. 6*d*., due $3\frac{3}{4}$ yr. hence, at 4%.

2. Find the discount on

(*a*) \$636, due in 9 mo. at 8%.
(*b*) \$1884.30, due in $3\frac{1}{5}$ yr. at 10%.
(*c*) \$637.50, due in $5\frac{1}{2}$ yr. at 5%.
(*d*) £1165 16*s*. 3*d*., due in $2\frac{1}{2}$ yr. at 6%.
(*e*) £252 19*s*. 3*d*., due in 9 mo. at $4\frac{1}{3}$%.

3. Find the present worth of \$6934.50, due 3 yr. hence, at 5%.

4. Find the discount on \$68.40, due $1\frac{1}{4}$ yr. hence, at $5\frac{1}{2}$%.

5. A tradesman accepts \$19.3125 in payment of a debt of \$$20\frac{267}{560}$, due in 12 mo., in consideration of being paid at once. What rate of discount does he allow?

6. Find the present worth of a bill for \$1127.10, drawn Jan. 1 at 4 mo., and discounted Feb. 20 at 10% per annum.

7. The discount on \$275 for a certain time is \$25. What is the discount on the same sum (*a*) for twice that time, and (*b*) for half that time?

8. A tradesman marks his goods with two prices, one for cash and the other for credit of 6 mo. What relation should the prices bear to each other, allowing interest at $7\frac{1}{2}\%$? If the credit price of an article be \$33.20, what is the cash price?

9. If \$98 be accepted in present payment of \$128, due some time hence, what should be the proper discount of a bill of \$128 which has only half the time to run?

10. A certain sum ought to have \$20.80 allowed as 8 mo. interest on it. But a bill for the same sum due in 8 mo. at the same rate should have \$20 only allowed off as discount in consideration of present payment. What is the sum and the rate per cent.?

EQUATION OF PAYMENTS

18. When several sums of money are due from A to B, payable at different times, it is often required to find the time, called the **Equated Time,** at which all may be paid together, without injustice to A or B.

When great exactness is demanded, interest must be added to the sums paid after they are due, and discount subtracted from the sums paid before they are due. But in practice the following Rule is sufficiently accurate :—

Multiply each debt by the number of days (or months) after which it is due. Add the results together. Divide this sum by the sum of the debts. The quotient will be the number of days (or months) in the equated time.

Take the following examples :—

Ex. 1. If \$300 be due from A to B at the end of 5 months, and \$700 at the end of 9 months, when may both sums be paid in a single payment without unfairness to A or to B?

$$\text{Number of months in equated time} = \frac{300 \times 5 + 700 \times 9}{300 + 700} = \frac{7800}{1000} = 7\tfrac{4}{5};$$

∴ the whole amount of the debt should be paid at the end of

The principle on which this solution depends is that the interest of the money, the payment of which is delayed beyond the time at which it is due, is equal to the interest of that which is to be paid before it becomes due.

In the above example $300 is kept $2\frac{4}{5}$ months after it is due, and the interest on it for that time is the same as the interest on $840, $(300 × $2\frac{4}{5}$) for one month.

But $700 is paid $1\frac{1}{5}$ months before it is due, and the interest on it for that time is the same as the interest on $840, $(700 × $1\frac{1}{5}$) for one month.

Ex. 2. *A* is indebted to *B* in the following amounts: $500, due in 6 months; $600, due in 7 months; $800, due in 10 months. Find the time when all these payments should be made together.

$$\begin{array}{rrr}
500 \times 6 = & & 3000 \\
600 \times 7 = & & 4200 \\
800 \times 10 = & & 8000 \\
\hline
1900 & 1900) & 15200 \\
\hline
 & & 8;
\end{array}$$

$\therefore$ the equated time $= 8$ mo.

NOTE.—*This method is but a rough approximation, and can only be taken as equitable when the various times of payment are not widely apart. It will, in short, be applicable only to cases which occur in the ordinary course of trade, and is therefore all that we require in the present work.*

It is also to be observed that the error involved in this method is *slightly in favor of the payer*, because interest is calculated on the payments made before they are due, instead of discount, in the algebraical process from which the method is derived.

Examples VIII

1. What is the equated time of $250 due 4 mo. hence, and $350 due 10 mo. hence?

2. Find the equated time of \$300 due in 3 mo. hence, \$400 due in 4 mo. hence, and \$500 due in 6 mo. hence.

3. On Jan. 15, I bought a bill of goods amounting to \$900, \$275 of which was on 30 da. credit, \$300 on 60 da., and \$325 on 90 da. On what date should the debt be discharged by one payment?

4. A debt of \$2400 was contracted on March 6, 1896, payable in 8 mo., but \$400 was paid in 2 mo., \$500 in 5 mo., \$800 in 7 mo. What was the equitable time for paying the balance?

5. The sum of \$1200 is due in 14 mo. If $\frac{1}{3}$ of the sum be paid in 9 mo., and $\frac{1}{5}$ of it in 13 mo., in what time ought the remainder to be paid?

6. One-half of a debt of \$1000 is due in 10 mo., $\frac{1}{4}$ of it in 12 mo., $\frac{1}{5}$ in 16 mo. and the remainder in 20 mo. When might the whole be paid at one payment?

7. A debt is due in 12 mo. hence, but $\frac{1}{9}$ of it is paid in 6 mo., and $\frac{1}{18}$ in 9 mo. When should the remainder be paid?

8. Of a debt of \$1400, \$100 is due immediately, \$600 at the end of 1 mo., \$400 at the end of 7 mo., and the remainder at the end of a year. At what time might the whole debt fairly be paid in one sum?

9. A grocer ought to receive from a customer \$50 at the end of 2 mo., \$30 at the end of 4 mo., and \$20 at the end of $6\frac{1}{2}$ mo. What would be the proper time for receiving the whole sum together?

10. A debt is to be paid as follows: One-sixth now, and one-sixth every 3 mo., until the whole is paid. When might the whole debt be paid at once?

EQUATION OF ACCOUNTS

19. Equation of Accounts (also called "Averaging of Accounts" and "Compound Equation of Payments") is the process of finding at what time the *balance of an account* can be paid without gain or loss to either party.

The **Balance of an Account** is the difference between the two sides of it, and is what one owes the other.

Ex. T. Black's Account in our Ledger.

Dr.		T. Black.			Cr.
1900.			1900.		
Jan. 1	To Mdse.	$500.00	Feb. 10	By Cash	$1000.00
Feb. 4	" "	600.00	Mar. 4	" "	600.00
Mar. 10	" "	800.00			

Take the latest date, Mar. 10, as focal date.

Due Dates.	Days.		When Paid.	Days.	
Jan. 1	500 × 68 =	34000	Feb. 10	1000 × 28 =	28000
Feb. 4	600 × 34 =	20400	Mar. 4	600 × 6 =	3600
Mar. 10	800 × 0 =	0			
	1900	54400		1600	31600
	1600	31600			
Bal. of ac.	300	22800	Bal. of products.		

Number of days = 22800 ÷ 300 = 76.
Count 76 days backward from Mar. 10;
∴ $300 is due Dec. 24, 1899.

The debit side shows the amount Black has received.
The credit side shows the amount Black has given.
Suppose the account to be settled on the latest date, March 10.

Dr.

Then, interest on			Interest on	
$500 for 68 da.	= $1	for 500 × 68 da., or	24000 da.	
600 " 34 "	= 1	" 600 × 34 " "	20400 "	
800 " 0 "	= 1	" 500 × 0 " "	0 "	
$1900	= 1	for	54400 "	

Cr.

Interest on   Interest on

$1000 for 28 da. = $1 for 1000 × 28 da., or 28000 da.

600 " 6 " = 1 " 600 × 6 " " 3600 "

$1600 = $1 for 31600 "

∴ Black receives $1900 and the int. on $1 for 54400 da.

" gives $1600 " " " " $1 " 31600 "

" owes $300 " " " " $1 " 22800 "

or " " $300 " " " " $300 " $\frac{22800}{300}$ " or 76 da.;

∴ $300 is due 76 days before March 10, or Dec. 24.

(a) *Find the date when each item is due or paid on both sides.*

(b) *Take the latest due date on either side, thus found, for the focal date. Multiply each item on both sides of the account by the number of days between the focal date and the date of the item.*

(c) *Add the products on each side, and subtract the sum of those on the one side from the sum of those on the other, and divide the difference by the balance of the account. The quotient is the number of days to be counted forward from the focal date, when the balance of the products and the balance of the account are on opposite sides, and backward from the focal date when they come on the same side.*

Examples IX

1. In the following account it is required to find the balance, and when it is due.

Dr. James Adamson. Cr.

1900.		$	1900.		$
Mar. 9	To merchandise..	300	Mar. 20	By cash..........	247
May 12	" " ..	474	April 11	" draft at 30 da..	400
June 19	" " ..	564	July 10	" cash	260

2. Find the equitable balance of the following account :—

DR. JOHN JONES IN ACCOUNT WITH WM. SMITH. CR.

1900.		$	1900.		$
Oct. 2	To merchandise....	180	Nov. 18	By mdse., 2 mo...	150
Nov. 8	" " 3 mo.	120	Dec. 24	" cash..........	200
Dec. 16	" " 4 mo.	240			

3. Find the equated time for paying the balance of the following account :—

DR. SCOTT, HUGHES & CO. CR.

1899.			1899.		
Jan. 29	Mdse., 2 m.	$519.00	Jan. 2	Mdse., 3 m.	$357.00
Feb. 5	Mdse., 3 m.	423.00	" 25	Mdse., 2 m.	738.00
" 19	Mdse., 2 m.	969.00	Feb. 4	Mdse., 3 m.	421.00

ALLIGATION

20. Alligation is the process by which we find the mean or average price of a compound when we mix or unite two or more articles of different values.

Ex. 1. A merchant has brown sugar worth 8 cents per pound, New Orleans worth 9 cents, and refined sugar worth 14 cents. How many pounds of each kind must he use in order to form a mixture worth 12 cents per pound?

By selling the mixture at 12c. per pound we see that 8c. (brown) gains 4c. on 1 lb.

∴ 1c. is gained on $\frac{1}{4}$ lb.

9c. (New Orleans) gains 3c. on 1 lb.;

∴ 1c. is gained on $\frac{1}{3}$ lb.

14c. (refined) loses 2c. on 1 lb.;

∴ 1c. is lost on $\frac{1}{2}$ lb.

Now with every cent *gain* he must combine a cent *loss*; hence he must have

$$\left.\begin{array}{l}\frac{1}{4}\text{ lb. at }\ 8\text{c.}\\ \frac{1}{2}\text{ lb. at }14\text{c.}\\ \frac{1}{3}\text{ lb. at }\ 9\text{c.}\\ \frac{1}{2}\text{ lb. at }14\text{c.}\end{array}\right\}=\left\{\begin{array}{l}3\text{ lb. at }\ 8\text{c.}\\ 6\text{ lb. at }14\text{c.}\\ 4\text{ lb. at }\ 9\text{c.}\\ 6\text{ lb. at }14\text{c.}\end{array}\right.$$

He must, therefore, have 3 lb. of brown sugar, 4 lb. New Orleans, and 12 lb. refined.

We may show that these quantities will make the mixture required, as follows:

$$\begin{array}{lll} 3\text{ lb. at }\ 8\text{c. per lb.} & = & 24\text{c.}\\ 4\text{ lb. at }\ 9\text{c.}\quad\text{"} & = & 36\text{c.}\\ \underline{12}\text{ lb. at }14\text{c.}\quad\text{"} & = & \underline{168\text{c.}}\\ 19\quad=\text{whole mixture.} & & 228\text{c.}=\text{value of mixture.}\end{array}$$

Hence, if 19 lb. be worth 228c.

1 lb. is worth $\frac{228}{19}=12$c.

Or we may reason thus: The 1c. *gained* on the $\frac{1}{4}$ lb. of brown exactly balances the 1c. *lost* on the $\frac{1}{2}$ lb. of the refined. Hence he must take $\frac{1}{4}$ lb. of the brown and $\frac{1}{2}$ lb. of the refined, or 2 lb. of one and 4 lb. of the other.

Similarly, for every 2 lb. of New Orleans there must be 3 lb. of refined. As 4 lb. of refined were required to balance the brown, and 3 lb. of the refined to balance the New Orleans, there must be 7 lb. of the refined in the compound. Therefore, the respective quantities are 2 lb. brown, 2 lb. New Orleans, and 7 lb. refined.

From the above we see that in examples of this kind a variety of answers may frequently be obtained, and all of them may be correct. To ascertain their correctness we resort to the method of proof given in this example.

From the above analysis we derive an easy practical method of solving such questions.

Ex. 2. How much sugar at 10, 13, 15, 17 and 18c. per pound must be taken to make a mixture worth 16c. per pound?

We proceed as follows :—

Differences.	16	
	—	
6	10	1
3	13	1
1	15	1
..	.	..
1	17	2, 4, 6, 8
2	18	4, 3, 2, 1

Write down the prices in a vertical column, and place the *differences* between these prices and the *mean* in a second vertical column to the left. Now take 1 at 10, 1 at 13, and 1 at 15 (the lowest that could be taken); this would represent a *loss* of 10 as compared with the *mean;* and this loss must be balanced by taking the necessary multiples of the differences, 1 and 2, which represent *gain*, as compared with the mean.

It is seen that this loss of 10 can be made up in *four* ways. By 2 at 17, 4 at 18; 4 at 17, 3 at 18; 6 at 17, 2 at 18; 8 at 17, and 1 at 18.

Other combinations may be made, as *e.g.*,

6	10	1
3	13	1
1	15	2
..	..	..
1	17	1, 3, 5, 7, 9
2	18	5, 4, 3, 2, 1

Here 1 at 10, 1 at 13, and 2 at 15, give *loss* of 11, which can be made up by multiples of the differences 1 and 2 (opposite 17 and 18) in *five* ways, as indicated.

Also,

6	10	1
3	13	2
1	15	1
.	..	..
1	17	1, 3, 5, 7, 9, 11
2	18	6, 5, 4, 3, 2, 1

When 1 at 10, 2 at 13, and 1 at 15 give 13 loss, which may be made up in *six* different ways.

Again,

6	10	2
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8, 10, 12, 1
2	18	7, 6, 5, 4, 3, 2, 1

Where 2 at 10, 1 at 13, and 1 at 15 give *loss* of 16, which may be made up in *seven* ways.

Also,

6	10	1
3	13	1
1	15	3
..	..	..
1	17	2, 4, 6, 8, 10
2	18	5, 4, 3, 2, 1

When 1 at 10, 1 at 13, and 3 at 15 give *loss* of 12, which may be made up in *five* ways; and thus an indefinite number of the combinations may be formed.

It should be observed that if the differences opposite the prices *less* than the *mean* are greater together than the sum of the other differences (as in the example), we assign numbers (the *lowest* possible) to the prices less than the mean FIRST, and *vice versa*; *e.g.* of the latter case.

Ex. 3. How much coffee at 25, 24, 23, 22, 21, 19, 18, and 17 cents per pound must be taken to make a mixture worth 20 cents per pound?

Diff's.	20	
3	17	4, 3, 2, 1, 1, 2
2	18	1, 2, 3, 5, 4, 2, etc.
1	19	1, 2, 3, 2, 4, 5
..	..	
1	21	1
2	22	1
3	23	1
4	24	1
5	25	1

Here the sum of the differences in excess of the mean is greater than that of the differences below the mean. We therefore assign *first* numbers to the prices which are greater than the mean, viz., 1 at 21, 1 at 22, 1 at 23, 1 at 24, 1 at 25. This gives a *gain* of 15, which may be balanced as above by 1 at 19, 1 at 18, and 4 at 17; or, by 2 at 19, 2 at 18 and 3 at 17, etc., etc.

Ex. 4. A grocer has 12 lb. of brown sugar, worth 10c. per pound, which he wishes to mix with clarified sugar worth 16c. per pound, so that the mixture may be worth 14c. per pound. How many pounds of clarified sugar must he take?

Proceeding as in the previous examples, without reference to the *quantity* of the brown sugar, we find that there must be 1 lb. brown sugar to 2 lb. clarified sugar. But as 12 lb.

of brown sugar are required, we must multiply each of these quantities by 12 in order that the gain and loss may be equal. We shall therefore have 12 × 2 lb. = 24 lb. of clarified sugar.

Ex. 5. A grocer wishes to mix 20 lb. sugar worth 9c. per pound and 10 lb. worth 12c. per pound, with clarified sugar worth 15c., so that the compound may sell at 13c. How much of the clarified must he take?

$$\begin{array}{llr}
20 \text{ lb. at} & 9\text{c.} = & \$1.80 \\
10 \text{ `` } & \text{`` } 12\text{c.} = & \$1.20 \\
\hline
30 & & \$3.00
\end{array}$$

Then, if 30 lb. is worth \$3,

1 " " $\$\frac{3}{30} = 10\text{c.}$

The value of 1 " of the mixture is, therefore, worth 10c.

The question may then be read as follows :—

How many pounds of clarified sugar worth 15c. per pound must be mixed with 30 lb. of another kind of sugar, worth 10c. per pound, so that the mixture may be sold for 13c. per pound?

The question in this form has already been fully explained.

Ex. 6. A merchant has West India sugar worth 8c. per pound, and New Orleans sugar worth 13c. He wishes to combine these so as to make a barrel containing 175 lb., which he may sell at 11c. per pound. How many pounds of each kind must he take?

Solving the question without reference to the 175 lb., we find that 2 lb. of West India sugar and 3 lb. of New Orleans sugar will form a mixture worth 11c. per pound. Adding these quantities we find that they form a mixture of 5 lb. But the required mixture is to contain 175 lb., or 35 times 5. We shall therefore have

35 × 2 lb. = 70 lb. West India sugar.
35 × 3 lb. = 105 lb. New Orleans sugar.

Examples X

1. A jeweler melted together 9 oz. of gold 22 carats fine, 12 oz. 18 carats fine, 9 oz. 21 carats fine. What is the fineness of the mixture?

2. If a merchant mixes 7 lb. of sugar worth 8c. a pound with 6 lb. worth 9c., 9 lb. worth 10c. and 10 lb. worth 12c., for how much must he sell the mixture to gain 25%?

3. On a certain day the thermometer ranged at 64° from 6 o'clock to 9, at 76° from 9 to 12, at 85° from 12 to 3, and at 68° from 3 to 6. What was the average temperature?

4. A person mixed 15 gal. of alcohol 80% strong, 12 gal. 90% strong, 23 gal. 60% strong, and 20 gal. 70% strong. What is the strength of the mixture?

5. What quantities of coffee, worth 23c. and 35c., respectively, per pound, must be mixed together so that the compound may be sold for 30c. per pound?

6. What quantity of oats at 35c. per bushel, rye at 60c. per bushel, and barley at 80c., must be taken to form a mixture worth 55c. per bushel?

7. How much tea, worth, respectively, 55c. and 75c. per pound, must be mixed with 30 lb. worth 90c. per pound, in order that the compound may be sold for 70c. per pound?

8. How much water will it require to dilute 60 gal. of alcohol, worth $1.50 per gallon, so that the mixture may be worth only $1.20 per gallon?

9. How many gallons of kerosene oil, worth 60c. per gallon, must be mixed with 12 gal. of coal oil, worth 36c., and 8 gal of Aurora oil, worth 56c., so that the compound may be sold for 50c. per gallon?

10. A farmer has 16 bu. of corn, worth 48c. per bushel, and 12 bu. of oats at 34c. per bushel, which he wishes to mix with rye at 60c. and barley at 80c., in order to sell the

compound at 56c. per bushel. How many bushels of rye and barley will be required?

11. A confectioner mixes three different qualities of candy worth, respectively, 14c., 18c., and 30c. per pound, so as to make a box of 84 lb. How many pounds of each sort must he take so as to sell the compound at an average price of 24c. per pound?

12. A farmer has three different qualities of wool, worth, respectively, 33c., 37c., and 45c. per pound. He wishes to make up a package amounting to 120 lb., which he can afford to sell at 39c. per pound. How many pounds of each kind must he take?

13. How many sheep worth, respectively, $1.50, $2, $2.75, $3, and $4 apiece, can be taken to make a flock of 300 worth $2.50 apiece?

14. How much tea at 40c. and 50c. per pound must be mixed with 36 lb. at 60c., so that the mixture may be sold at 77c., at a gain of 40%?

15. How much tea at 30c., 35c., 40c., 45c., and 50c. must be taken to form a mixture of 100 lb. at 56c., so as to gain $33\frac{1}{3}$%?

16. A farmer wishes to mix corn worth 70c. per bushel with rye worth 75c., barley worth 60c., and oats worth 45c., to make a mixture of 60 bu., which he may sell at 78c. per bushel, at a gain of 20%. How many bushels of each kind must he take?

17. If 16 gal. of spirits at $1.25 per gallon are mixed with 9 gal. at a different price, and 25% is gained by selling a gallon of the mixture at $1.67½, what is the price of the second kind of spirits per gallon?

18. A man paid $165 to 55 laborers, consisting of men, women and boys. To the men he paid $5 per week, to the women $1 per week, and to the boys $½ per week. How many were there of each?

19. A man bought calves, sheep, and lambs, 154 in all, for \$154. He paid \$3½ for each calf, \$1⅓ for each sheep, and \$½ for each lamb. How many did he buy of each kind?

20. A farmer bought 100 animals for \$100. Geese at \$½ each, pigs at \$3½, and calves at \$10. How many animals were there of each kind?

21. A grocer has three kinds of tea; the second being ½ as dear again as the first, and the third ¼ as dear again as the second. If he mixes a certain quantity of the first with twice as much of the second and 112 lb. of the third, and finds the mixture to be 1/[illegible] as dear again as the second sort, of how many pounds does the mixture consist?

ANSWERS TO EXAMPLES IN THE SUPPLEMENT

Examples I. Page 201

1. 16.   2. 32.   3. 42.   4. 79.   5. 85.   6. 64.
7. 34.   8. 73.   9. 92.   10. 99.   11. 39.   12. 63.

Examples II. Page 202

1. 245.   2. 531.   3. 307.   4. 670.   5. 128
6. 179.   7. 463.   8. 103.   9. 256.   10. 579.
11. 438.   12. 507.   13. 686.   14. 708.   15. 888.
16. 512.   17. 4968.   18. 8765.

Examples III. Page 203

1. .73.   2. .364.   3. 30.02.   4. $\frac{11}{12}$.   5. $\frac{5}{7}$.
6. $1\frac{5}{7}$.   7. $7\frac{2}{5}$.   8. 1.709.   9. 8.320.   10. .495.
11. 2.516.   12. .822.   13. .908.   14. .693.   15. 1.966.
16. 1.473.

Examples IV. Page 204

1. 27.   2. 45.   3. 6.3.   4. 13.   5. 54.   6. 8.1.

Examples V. Page 204

1. 879.   2. 3420.   3. 93, 31, 62.
4. 55, 110, 220, 440.   5. 35856144, 35832196.   7. 1512.   8. 9.

EXAMPLES VI. PAGE 206

1. 2.2894. 2. 3.1072. 3. 6.0822. 4. 7.2112. 5. .9654.
6. .4308. 7. 2.1006. 8. .8372. 9. 2.0614.

EXAMPLES VII. PAGE 210

1. (*a*) $4600; (*b*) $70; (*c*) $562.50; (*d*) $1050; (*e*) £537 10*s*.
2. (*a*) $36; (*b*) $456.80; (*c*) $137.50; (*d*) [illegible] 1*s*. 3*d*.;
(*e*) £7 19*s*. 3*d*. 3. $6030. 4. $4.40.
5. 5%. 6. 1105. 7. \$$45\frac{5}{8}$, \$$13\frac{2}{21}$.
8. 80 to 83; $32. 9. \$$16\frac{112}{113}$. 10. $520; 6%.

EXAMPLES VIII. PAGE 212

1. $7\frac{1}{2}$ mo. 2. $4\frac{7}{12}$ mo. 3. March 18.
4. $8\frac{1}{3}$ mo. after Nov. 6. 5. 4 mo. after 14 mo. 6. $12\frac{1}{5}$ mo.
7. 13 mo. 8. 5 mo. 9. $3\frac{1}{2}$ mo.
10. $7\frac{1}{2}$ mo.

EXAMPLES IX. PAGE 215

1. $431; May 9. 2. $190; March 11. 3. $395; May 28.

EXAMPLES X. PAGE 221

1. 18.825 carats. 2. $12\frac{1}{2}$c. 3. 73.25°. 4. $72\frac{2}{7}$%.
5. 5 lb. of first, 7 lb. of second.
6. 30 bu. oats; 20 bu. rye; 20 bu. barley.
7. 50 lb. at 55c.; 30 lb. at 75c. 8. 15 gal. water.
9. 12 gal. kerosene. 10. 14 bu. rye; 14 bu. barley.
11. 18 lb. at 14c.; 18 lb. at 18c.; 48 lb. at 30c.
12. 36 lb. at 33c.; 36 lb. at 37c.; 48 lb. at 45c.
13. 60 at $1.50, 60 at $2, 120 at $2.75, 30 at $3, and 30 at $4.
14. 9 lb. at 40c., and 9 lb. at 50c.
15. 10 lb. at 30c., 10 lb. at 35c., 20 lb. at 40c., 50 lb. at 45c. and 10 lb. at 50c.
16. 12 bu. corn, 24 bu. rye, 12 bu. barley, 12 bu. oats.
17. $1.50. 18. 30 men, 5 women, 20 boys.
19. 14 calves, 42 sheep, 98 lambs.
20. 92 geese, 4 pigs, and 4 calves. 21. 130 lb.